

Proportion Problems

Comparing *ratios* of related items.

Q = Quantity (unknown)
K = Kwantity (given)
q = quantity (given)
k = kwantity (given)

Kwantity & kwantity
are made-up words.

$$\frac{Q}{K} = \frac{q}{k}$$

[kwik-kwik]

Shoot-the-Chute

$$\frac{Q}{K} = \frac{q}{k}$$

Q & q have the same units.
K & k have the same units.

K and k units
dissolve,
leaving only
Q = q units.

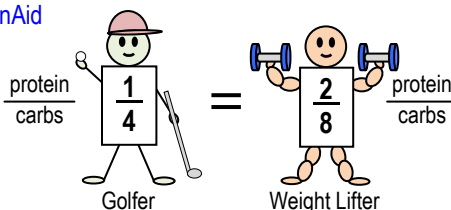
- Given K, q, & k, set up and solve for Q using Shoot-the-Chute.
- For relations of 3 or more items, use a Q/K = q/uac/k chain.
- Q/K=q/k is the parent of Conversion, Q=RK, and Q=PK problems.



Equivalent Pro Portions

RATIO
relation
fraction

PROPORTION
ratio = ratio

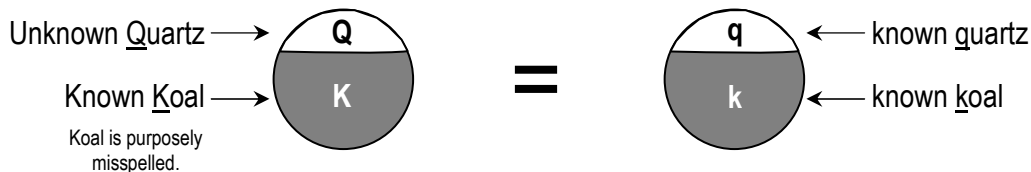


EQUIVALENT
A
L
U
E
Equal in
value, not in
appearance.

Imagine that pro athletes eat rations of nutrients in the same pro portions: 1 part protein to 4 parts carbs. Athletes in light sports need fewer calories so eat smaller portions than those in heavy sports.



Shoot-the-Chute

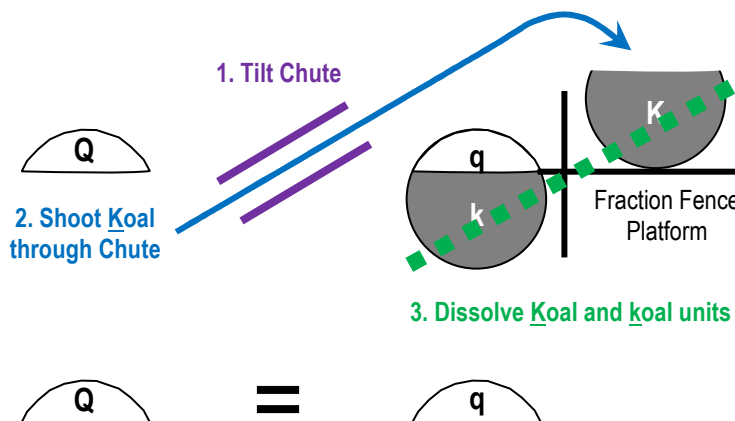


Shoot-The-Chute
is a shortcut for moving
items to opposite sides
of the equal sign.

In this case, it's
equivalent to multiplying
both sides by K.

$$\cancel{(K)} \frac{Q}{\cancel{K}} = \frac{q}{k} \cancel{(K)}$$

$$Q = q$$



Advantages
Q/K=q/k provides a
template for
consistent problem
set up.

Shoot-the-Chute
minimizes algebraic
manipulation.

$$Q/K = q/k$$

Relations of 2 items [kwik-kwik]

$$\frac{Q}{K} = \frac{q}{k} \quad \leftarrow \text{Fraction Fence}$$

It takes 2 cups to feed 4 kids. How many cups are needed to feed 6 kids?

2 items
* cups
* kids

Find cups
to feed
6 kids...

Set Up

$$\frac{Q \text{ cups}}{6 \text{ kids}} = \frac{2 \text{ cups}}{4 \text{ kids}}$$

...given that
2 cups feed
4 kids.

Observe
"cups" on top,
"kids" on bottom.

Shoot-the-Chute

Draw the "shoot" line *through*
"6 kids" to show it's gone.

$$\frac{Q \text{ cups}}{\cancel{6 \text{ kids}}} = \frac{2 \text{ cups}}{4 \text{ kids}} \quad \bigg| \quad 6 \text{ kids}$$

Extend the Fraction
Fence so that "6 kids"
has a landing platform.

Dissolve Units

$$\frac{Q \text{ cups}}{\cancel{6 \text{ kids}}} = \frac{2 \text{ cups}}{4 \text{ kids}} \quad \bigg| \quad \cancel{6 \text{ kids}}$$

Dissolve "kids" units.
Only "cups" remain,
which is the goal.

Do the Math

$$\frac{Q \text{ cups}}{\cancel{6 \text{ kids}}} = \frac{2 \text{ cups}}{4 \text{ kids}} \quad \bigg| \quad \cancel{6 \text{ kids}}$$

Divide vertically and/or
diagonally when possible,
then multiply across.

$$Q = 3 \text{ cups}$$

Circle or box
the solution.

If three boxes hold 18 pens, how many pens do 7 boxes hold?

2 items
* pens
* boxes

$$\frac{Q \text{ pens}}{7 \text{ boxes}} = \frac{18 \text{ pens}}{3 \text{ boxes}} \quad \bigg| \quad 7 \text{ boxes}$$

$$Q = 42 \text{ pens}$$

2 items require a
2-section fence.

Q/K = q/uac/k

Relations of 3 or more items [kwik-kwak]

$$\frac{Q}{K} = \frac{q}{u} \frac{u}{a} \frac{a}{c} \frac{c}{k}$$

Extended Fraction Fence

Set up and dissolve intermediate units.

If three boxes hold 18 pens and two cases contain 20 boxes, how many pens are in 5 cases?

3 items
* pens
* boxes
* cases

Match top units: 1st fence section

$$\frac{Q \text{ pens}}{5 \text{ cases}} = \frac{18 \text{ pens}}{3 \text{ boxes}} \frac{20 \text{ boxes}}{2 \text{ cases}}$$

Match bottom units: 2^{nd-to-last} fence section.

$$\frac{Q \text{ pens}}{5 \text{ cases}} = \frac{18 \text{ pens}}{3 \text{ boxes}} \frac{20 \text{ boxes}}{2 \text{ cases}} \frac{5 \text{ cases}}{1}$$

6 → 10

$$\frac{Q \text{ pens}}{5 \text{ cases}} = \frac{18 \text{ pens}}{3 \text{ boxes}} \frac{20 \text{ boxes}}{2 \text{ cases}} \frac{5 \text{ cases}}{1}$$

1 1

Q = 300 pens

3 items require a 3-section fence.

How many pens are in 2 trucks if there are 25 pens per box, 4 boxes per case, 10 cases per bin, and 50 bins per truck?

5 items
* pens
* boxes
* cases
* bins
* trucks

Match top units: 1st fence section

$$\frac{Q \text{ pens}}{2 \text{ trucks}} = \frac{25 \text{ pens}}{1 \text{ box}} \frac{4 \text{ boxes}}{1 \text{ case}} \frac{10 \text{ cases}}{1 \text{ bin}} \frac{50 \text{ bins}}{1 \text{ truck}} \frac{2 \text{ trucks}}{1}$$

Match bottom units: 2^{nd-to-last} fence section.

100 → 1,000 → 50,000 → 100,000

Q = 100,000 pens

5 items require a 5-section fence.

Proportion Problem Traps


Proportion Total Trap

A private school enrolls 3 boys for every 5 girls.
If there are 80 kids, how many are boys? How many are girls?

WRONG!

$$\frac{Q \text{ boys}}{80 \text{ kids}} = \frac{3 \text{ boys}}{5 \text{ kids}}$$

Trap: 5 = girls, *not* total kids.



$$\frac{Q \text{ boys}}{80 \text{ kids}} = \frac{3 \text{ boys}}{5 \text{ kids}} \rightarrow \frac{Q}{80} = \frac{3}{5} \rightarrow Q = 48 \text{ boys}$$

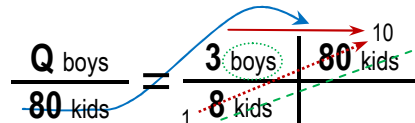
$$80 - 48 = 32 \text{ girls}$$

$Q = 48 \text{ boys}$
 $80 - 48 = 32 \text{ girls}$
 48:32 \neq 3:5
 There should be fewer boys than girls!

CORRECT

$$\frac{Q \text{ boys}}{80 \text{ kids}} = \frac{3 \text{ boys}}{8 \text{ kids}}$$

Calculate total kids from the 3:5 ratio:
3 boys + 5 girls = 8 kids



$$\frac{Q \text{ boys}}{80 \text{ kids}} = \frac{3 \text{ boys}}{8 \text{ kids}} \rightarrow \frac{Q}{80} = \frac{3}{8} \rightarrow Q = 30 \text{ boys}$$

$$80 - 30 = 50 \text{ girls}$$

$Q = 30 \text{ boys}$
 $80 - 30 = 50 \text{ girls}$
 30:50 = 3:5
 The correct ratio of boys to girls.

Inversely Proportional Traps

A biker rides at an average rate of 4 miles per hour (mph) for 8 hours.
What rate of speed must he ride to cover the *same* distance in 2 hours?

WRONG!

$$\frac{Q \text{ mph}}{2 \text{ hrs}} = \frac{4 \text{ mph}}{8 \text{ hrs}}$$

Trap: mph and hrs are *inversely* proportional.

$$Q = 1 \text{ mph}$$

He must ride *faster* to cover the same distance in less time!

CORRECT

$$D_1 = D_2$$

$$R_1 T_1 = R_2 T_2$$

$$4(8) = R_2(2)$$

$$16 \text{ mph} = R_2$$

See the Q=RK Rate Problems lesson for further explanation.

EXPLANATION

Q/K = q/k works for items that are *directly* proportional.

In Q=RK problems, **Rate** and **Kwantity** are *inversely* proportional. For a fixed **Quantity**, as **Rate** increases, **Kwantity** decreases, and vice versa.

$$Q = RK$$

Directly Proportional

- Quantity & Rate
 - Quantity & Kwantity
- Solve as Q/K=q/k or Q=RK.

Inversely Proportional

- Rate & Kwantity
- Solve as Q=RK.

See the Problem Analysis lesson for further explanation.

Note: In Q/K=q/k, Q is the unknown. In this problem, Q=Rate. In Q=RK, Q is the *product* of Rate & Kwantity.

10% of a 40 oz rock is gold. What percent of an 80 oz rock would yield the same amount of gold?

WRONG!

$$\frac{Q \%}{80 \text{ oz}} = \frac{10 \%}{40 \text{ oz}}$$

Trap: gold % and rock size are *inversely* proportional.

$$Q = 20 \%$$

For a larger rock, a *smaller* % of it would yield the same gold.

CORRECT

$$Q_1 = Q_2$$

$$P_1 K_1 = P_2 K_2$$

$$10\%(40) = P_2(80)$$

$$5\% = P_2$$

See the Q=PK Percent Problems lesson for further explanation.

EXPLANATION

Q/K = q/k works for items that are *directly* proportional.

In Q=PK problems, **Percent** & **Kwantity** are *inversely* proportional. For a fixed **Quantity**, as **Percent** increases, **Kwantity** decreases, and vice versa.

$$Q = PK$$

Directly Proportional

- Quantity & Percent
 - Quantity & Kwantity
- Solve as Q/K=q/k or Q=PK.

Inversely Proportional

- Percent & Kwantity
- Solve as Q=PK.

See the Problem Analysis lesson for further explanation.

Note: In Q/K=q/k, Q is the unknown. In this problem, Q=Percent. In Q=PK, Q is the *product* of Percent & Kwantity.



Your Turn!



True or False

- 1) _____ A ratio is a relation between numbers.
- 2) _____ Equivalent ratios form a proportion.
- 3) _____ With $Q/K=q/k$, you set up and solve proportions the same way each time.
- 4) _____ Shoot-the-Chute reduces the steps needed to isolate the unknown variable.
- 5) _____ $Q/K=q/k$ works for inversely proportional items.
- 6) A jar of instant coffee makes 20 cups. How many cups would 4 jars make?
- 7) A jar of instant coffee makes 20 cups. How many jars are needed to make 60 cups?
- 8) A pack of gum has 10 sticks. A box has 20 packs. How many sticks are in 4 boxes?
- 9) A pack has 10 sticks; a box has 20 packs; a crate has 15 boxes. How many sticks are in 5 crates?
- 10) For every 4 apples, John picks 3 oranges. If he picks 70 pieces of fruit, how many are apples?

Answers: 1T, 2T, 3T, 4T, 5F, 6) 80 cups, 7) 3 jars, 8) 800 sticks, 9) 15000 sticks, 10) 40 apples