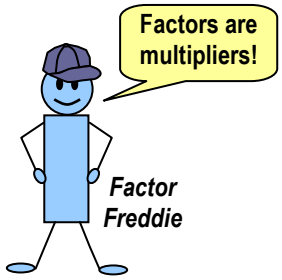


Factors

Factors are integers multiplied to make Products.

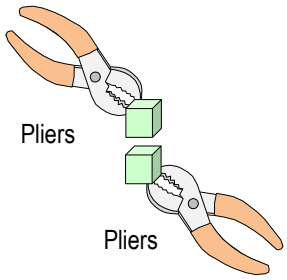
Factor × Factor = Product

2 × 3 = 6

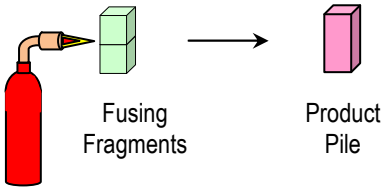
Paradox!
Factors are also ***divisors*** of the Products they make.

BrainAids

Factors are *Multi-Pliers* used to make Products.

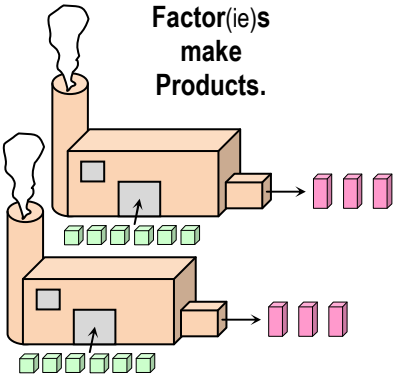


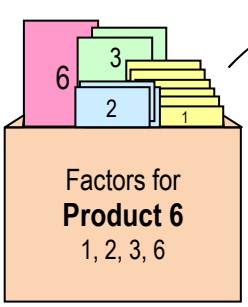
Factor Fragments make Product Piles!
Imagine stacking fragments into a pile and fusing them into a finished product.



(Caution: Factors are integers, *not* fractions!)

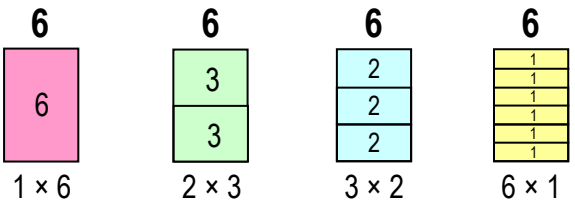
Factor(ie)s make Products.





Factors for **Product 6**
1, 2, 3, 6

Imagine the factors used to make a product are in a box waiting for assembly.



Product 6 can be assembled using various factors.

Can Factors be Negative?
In standard arithmetic, factors are generally considered to be *positive* integers (1, 2, 3...). However, when it comes to algebra, *negative* integers (…-3, -2, -1) can also be factors.

Factor as Noun
2 is a *factor* of the product 6.

Factor as Verb
Factor the product 6.

Factoring
The process of extracting (*all*, *prime*, or *common*) factors from products.

TRAP

Zero = Factor
Zero ≠ Divisor

Zero can be a factor since zero times any other integer yields a product, namely zero.

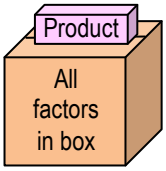
But in standard math, division by zero is not permitted, so zero can *not* be a Divisor.

TRAP

Fractions ≠ Factors

Fractions *can* be used as multipliers to make products:
[½ × 6 = 3].

However, by definition, fractions are *not* considered to be factors.



Extracting *All* Factors

All-In-The-Box Factoring

To extract ALL factors, start with 1 and continue in numerical order: 2, 3, 4, 5…

Product sits on top of box lid.

Product
factor pair
factor pair
factor pair

Arrow forms bottom of box.

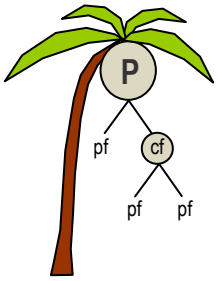
All pairs of factors fit inside the box.

Tip Use the *Divisibility Dreams* shortcuts to predict if a number is a factor.

Draw box lid and place desired Product on top.	Start with 1 and the Product itself as the first *factor pair.*	If 2 is a factor, list its pair. If 3 is a factor, list its pair. Proceed in 4, 5, 6… order. Skip non-factors.	Stop at or erase the first repeated pair.	Draw an arrow to "box" the factors, then follow the arrow to list them from low to high.
6	**6** 1 × 6	**6** 1 × 6 2 × 3	**6** 1 × 6 2 × 3 ~~3 × 2~~	**6** 1 × 6 2 × 3 **All Factors** 1, 2, 3, 6
12	**12** 1 × 12	**12** 1 × 12 2 × 6	**12** 1 × 12 2 × 6 3 × 4 ~~4 × 3~~	**12** 1 × 12 2 × 6 3 × 4 **All Factors** 1, 2, 3, 4, 6, 12
32	**32** 1 × 32	**32** 1 × 32 2 × 16	**32** 1 × 32 2 × 16 4 × 8 ~~8 × 4~~	**32** 1 × 32 2 × 16 4 × 8 **All Factors** 1, 2, 4, 8, 16, 32

Extracting *Prime* Factors

Prime Coconut Factoring



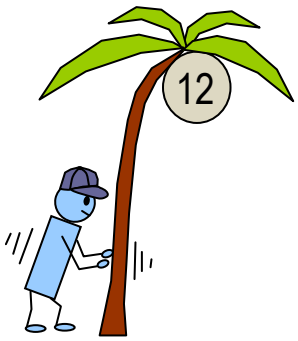
Imagine that the Product is a falling coconut that splits into prime factor and composite factor nutrients.

To extract PRIME factors, start with 2 and continue in prime number order: 3, 5, 7, 11, 13…

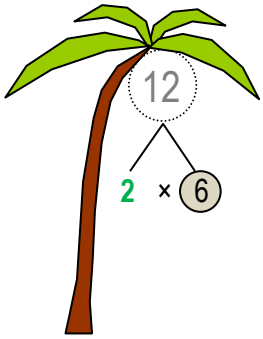
Tip

Use the *Divisibility Dreams* shortcuts to predict if a prime number is a factor.

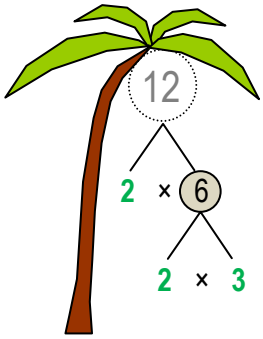
Place the Product in a coconut and shake its tree to dislodge it.



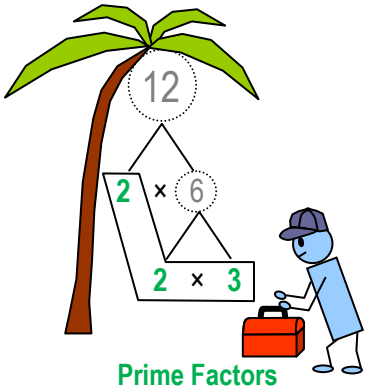
Proceed in 2, 3, 5, 7… order to find and split out the first factor pair.



Proceed in 2, 3, 5, 7… order to split any composites into primes.



Box the prime factors to eat later for lunch.



Prime Factors

$2 \times 2 \times 3$ -or- $2^2 \times 3$

Check 4

12 √

Using exponent

16

2 × 8

2 × 4

2 × 2

2^4

27

3 × 9

3 × 3

3^3

35

5 × 7

47

1 × 47

Already Prime!

72 vs. **72**

2 × 36

2 × 18

2 × 9

3 × 3

8 × 9

2 × 4 3 × 3

2 × 2

$2^3 \times 3^2$

If you choose to first split a product into its times-table factors, take care to box *all* prime factors, which may *not* be in order.

144

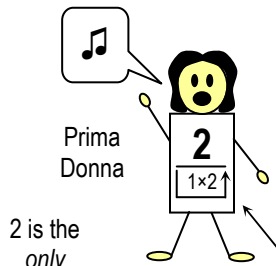
12 × 12

3 × 4 3 × 4

2 × 2 2 × 2

$2^4 \times 3^2$

Prime vs. Composite Numbers

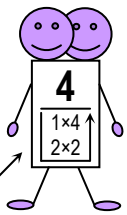


2 is the *only* even prime!

Prime
Exactly divisible by 1 and itself only.

0 and 1
Neither prime nor composite!

Composite
Exactly divisible by 1 and itself and at least one other number.



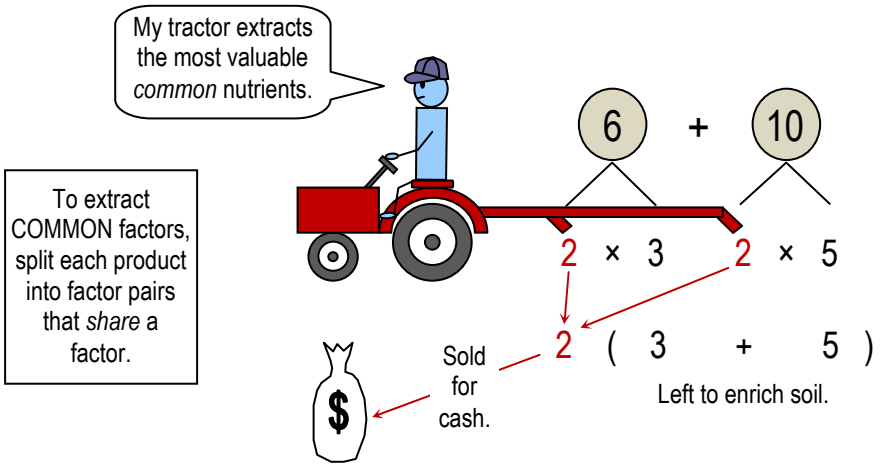
Prime Test
If All-In-The-Box Factoring yields *one* factor pair.

Composite Test
If All-In-The-Box Factoring yields *multiple* factor pairs.

Composed of many!

Extracting *Common* Factors

Common Factor ExTractor



To extract COMMON factors, split each product into factor pairs that *share* a factor.

6 + 10

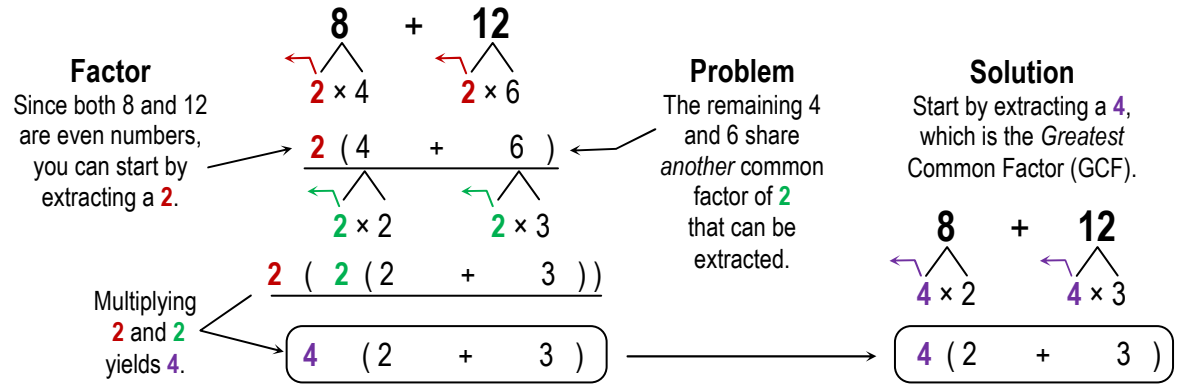
16

is equivalent to

2 (3 + 5)

2 (8)

16



Factor
Since both 8 and 12 are even numbers, you can start by extracting a **2**.

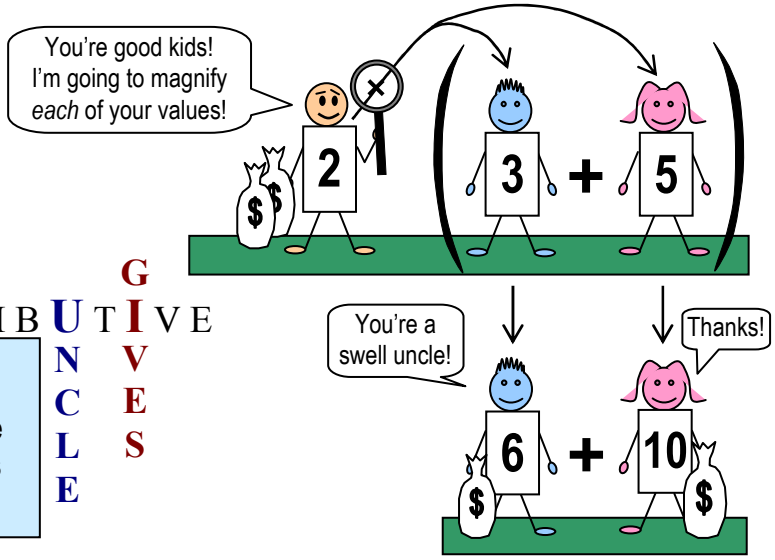
Problem
The remaining 4 and 6 share *another* common factor of **2** that can be extracted.

Solution
Start by extracting a **4**, which is the *Greatest* Common Factor (GCF).

Multiplying **2** and **2** yields **4**.

Factoring vs. Distribution

Factoring is the opposite of the Distributive Property.



DISTRIBUTIVE GIVES UNCLE

Rich uncle distributes riches!

2 (3 + 5)

6 + 10

16

GCF: Greatest Common Factor

The GCF is the *largest* factor shared by the given products.

All-In-The-Box GCF

8

1 × 8
2 × 4

12

1 × 12
2 × 6
3 × 4

Circle factors that are common to each product.

Factors of 8: 1, 2, 4, 8
Factors of 12: 1, 2, 3, 4, 6, 12

Common Factors
1, 2, 4

GCF = 4

Greatest Common Factor of products 8 and 12 is **4**.

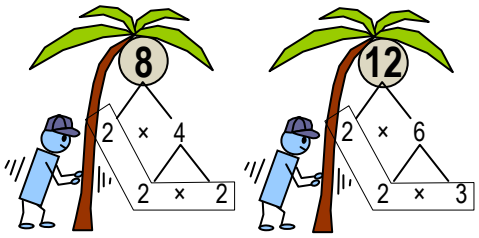
Paradox!
The ***Greatest*** Common Factor is ***smaller*** than the products it's derived from!

This holds true for positive GCFs. However, if the smaller product is a factor of the larger product, the GCF *equals* the smaller product. Example: For products 2 and 4, the GCF = 2.

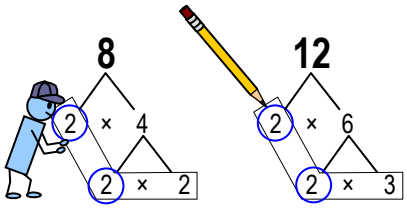
Prime Coconut GCF

Grip, Catch, Focus

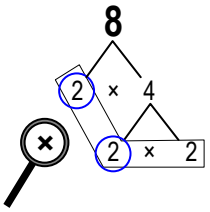
Grip each product tree and shake out and box its prime factors.



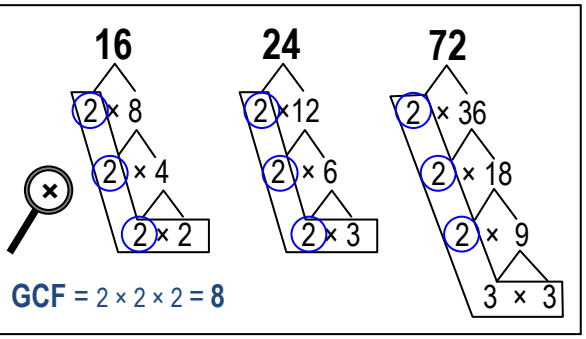
Catch (circle) each falling factor that is common to the products *each* time it occurs.



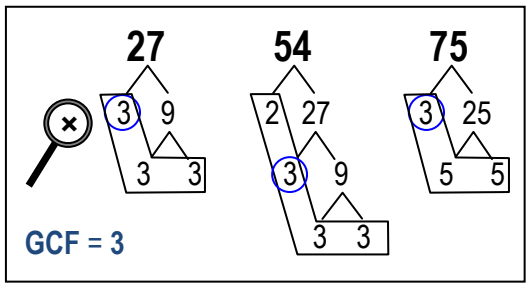
Focus on and magnify (multiply) *one* set of circled factors.



GCF = 2 × 2 = **4**

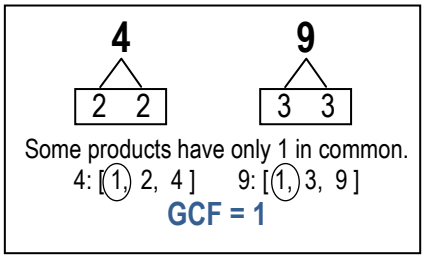


GCF = 2 × 2 × 2 = **8**



GCF = **3**

Tip Once you become familiar with prime factoring, it saves time to omit the × signs.



Some products have only 1 in common.
4: [1, 2, 4] 9: [1, 3, 9]
GCF = 1

Why Factor?

To Reduce Fractions

Divide numerator and denominator by a common factor (Division Diet).

Common Factoring
May not yield lowest terms.

$$\frac{34}{102} \div 2 = \frac{17}{51}$$

34 and 102 are even, so you divide by 2.

TRAP

51 looks prime, so you stop reducing.

Prime Factoring
Ensures lowest terms.

34 = 2 × 17

102 = 2 × 51, 3 × 17

$$\frac{34}{102} = \frac{2 \cdot 17}{2 \cdot 3 \cdot 17} = \frac{1}{3}$$

Per *Divisibility Dreams*, the S.O.D. of 51 = 5+1 = 6 which is divisible by 3.

To Simplify Expressions

Extracting common factors can reduce expressions to their simplest form.

$$\frac{2x + 4y}{3x + 6y} = \frac{2(x + 2y)}{3(x + 2y)} = \frac{2}{3}$$

To Solve Equations

Example: One way to start solving a Quadratic trinomial is to factor it into two binomials.

$$x^2 + 3x + 2 = 0$$

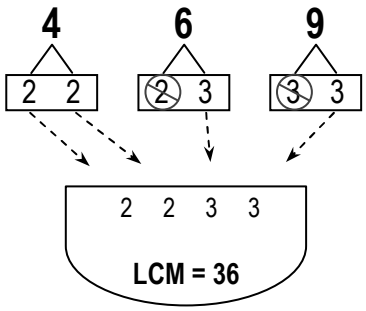
$$(x + 1)(x + 2)$$

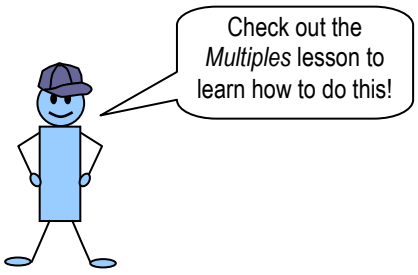
$x + 1 = 0$ $\quad$ $x + 2 = 0$

$x = -1$ $\quad$ $x = -2$

To Find the LCM

Prime factoring can be used as the first step in finding the LCM (Least Common Multiple).





Your Turn!

Matching

1) ___ Factor	a. Extracting multipliers from products.
2) ___ Product	b. Multipliers that make a Product.
3) ___ Factoring	c. Result of multiplying factors.
4) ___ Factor pair	d. Divisible by 1 and itself only.
5) ___ Prime factor	e. Multiplier

True or False

6) _____ Factors can be fractions.

7) _____ Factors can be negative.

8) _____ The word "factor" can be used as a noun or a verb.

9) _____ Factoring is the opposite of the Associative Property.

10) _____ A positive GCF is larger than the products it's derived from.

11) Extract all factors of 18.

12) Extract prime factors of 18.

13) Extract common factor of (15 + 25).

14) Find the GCF of 18 and 36.

Answers: 1e, 2c, 3a, 4b, 5d, 6F, 7T, 8T, 9F, 10F, 11)1,2,3,6,9,18, 12)2×3×3, 13)5(3+5), 14)18.