Max Learning's Fraction Fun

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Welcome!

Hi, my name is Max Learning, and I'll be your teacher and guide.

My goal is to make math seem "real" to you, so you'll gain confidence and *look forward* to your next math challenge.

The fact that you're reading this book means you're eager to succeed and are willing to explore new ways to do so. *So let's get started!*

Why Is Math A Struggle?	How This Book Can Help
Symbols Math uses symbols, *lots* of them. It's as difficult to learn as a foreign language.	**Mental Manipulatives** You'll learn to "see" three-dimensional objects behind each symbol.
Rules Math is based on rules, *lots* of them. It's hard not to confuse one for the other.	**BrainAids** You'll learn clever memory hints that make the rules easy and fun.
Trauma Getting an answer wrong in front of the class, losing at a flash-card competition, failing a test, being criticized by a teacher—all can lead to math trauma.	**RUFF** You'll learn to be in a Relaxed, Uncluttered, Focused, and Flowing state of mind, which increases confidence and eases past traumas.

What's Good About Math?

Certainty

Math problems have *right* answers. An essay you wrote for English class, or a project you made for Art class, might seem fabulous to you, but maybe not to your teachers. However, in math, when you get the right answer, no one can argue with it.

Quest

Math problems are puzzles. The quest to solve them can be exciting! Math can be more fun than any game you'll ever play. If math becomes fun, you'll look forward to, rather than run from, it.

Magic

Math is the *language of nature*. Nearly everything we see, hear, touch and do can be described with math. Every image, color, and sound on a computer is based on math. In today's movies, you can't always tell what's real and what's been generated by some mathematical formula. In short, math is amazing—there's magic in it!

Note to Parents

Although I've purposely kept the problems in this book relatively simple, parents of younger children, or of children who have trouble interpreting the text, may wish to first learn the techniques themselves, and then teach them to their children.

You're learning a new, I hope, more interesting way of doing fraction problems. As with learning anything new, it's best not to rush; so relax, take your time, and enjoy the process.

This is a *techniques* book rather than a *drill & practice* book. Check your answers to the **Your turn** activities in the **Answer Key** in the back of the book. Then apply these techniques to the numerous problems in traditional math textbooks, or make up some of your own.

Pronunciation Guide

Sometimes it may not be obvious how to pronounce terms you have not heard spoken. When you see a term followed by a pronunciation, refer to this guide as needed.

Vowels		
Long	**Short**	**Other**
aa = ate	a = act	ai=air, ar=are, aw=paw
ee = eel	e/eh = end	
ii = hi	i/ih = hid	
oh = no	aw = on	oo = book, or = for
		ow = how, oy = boy
yu = use	u/uh = up	uu = too, ur = fur

Consonants	
Hard	**Soft**
k = cat	s = ice
g = go	j = gem
s/ss = hiss	z = his
ch = chin	sh=shin; zh=vision
th = thin	thh = this
Accent on: UP-ur-KAASS	

Common Abbreviations

aka = also known as
e.g. = for example (think egzample)
i.e. = that is

BrainAids

It was a mouthful to say *mnemonic* (nee-MAWN-ik) *device,* so I coined the word *BrainAid* for memory hints that help you learn. Feel free to make up your own BrainAids. Most fall into one of the following "A" categories:

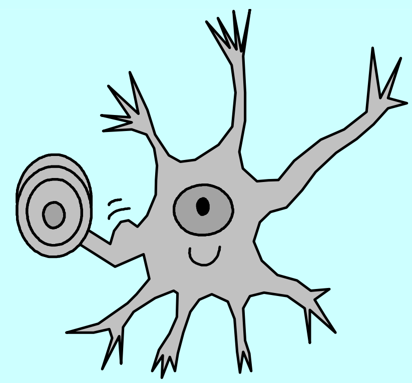
Analogy = Comparison

How to say it: uh-NOWL-uh-jee

What it is: A *comparison* of what you are trying to learn to what you already know.

Why it works: To learn, new brain fibers must grow and laboriously push their way through dense brain tissue, which can be very tiring. An analogy lets you piggyback new knowledge on top of *existing* brain fibers, which is quicker and takes much less effort.

Analogy Example: Just as *physical* exercise builds new *muscle* fibers, *mental* exercise builds new *brain* fibers. Both take time, effort, and repetition.



Analogy: Building brain fibers.

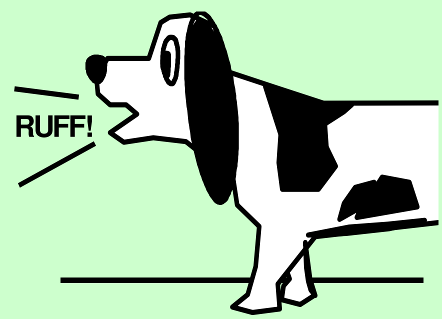
Acronym = Name

How to say it: AK-roh-nim

What it is: A *name* made from the first letters of several words. Hint: Think *nym* = *name*.

Why it works: The letters of an acronym act like small hooks on which you can hang large words. You memorize a little to remember a lot.

Acronym Example: To maximize your learning, be in a learning frame of mind: Relaxed, Uncluttered, Focused, and Flowing. In other words, be RUFF.



Acronym: RUFF

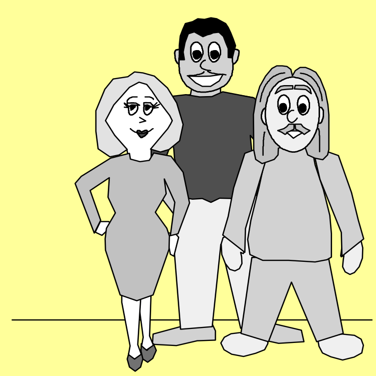
Acrostic = Story

How to say it: uh-KRAW-stik

What it is: A *story* made from the first letters of several words. Hint: Think *stic* = *st*ory.

Why it works: Sometimes a "story" is easier to remember than an acronym, especially if the acronym is hard to pronounce.

Acrostic Example: You create the acronym MTF to help you remember the names of three new acquaintances, Mary, Tom, and Fred. But MTF is hard to pronounce, so you convert it into the acrostic "My Three Friends."



Acrostic: My Three Friends

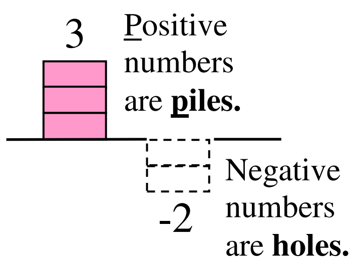
Concepts

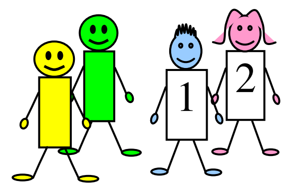
Mental Math Basics

In *Max Learning's Mental Math*, we learned several concepts that will help us in *Fraction Fun.*

Mental Manipulatives

Traditional manipulatives are physical objects, like tiles or blocks, which you "manipulate" to mimic math operations. *Mental* manipulatives are items you visualize when you see a number or operation. They can turn lifeless symbols into reality—at least in your imagination. And unlike physical manipulatives, mental manipulatives are always with you to make math more engaging.





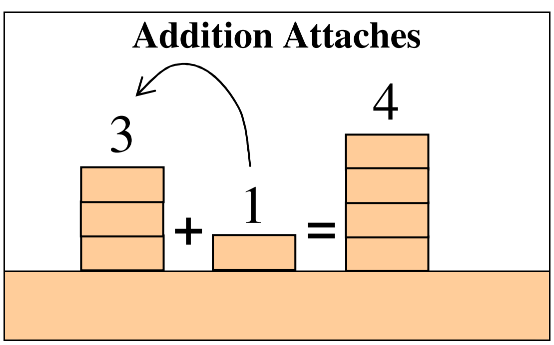
MathBots manipulate piles and holes or represent numbers.

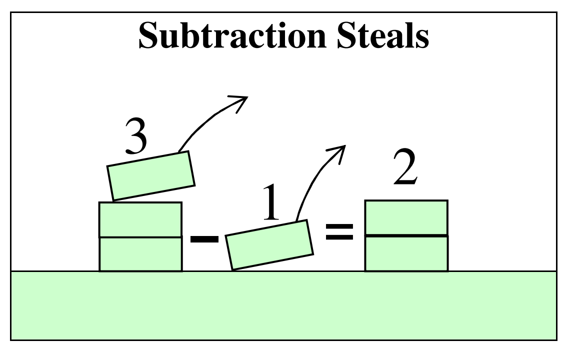
Numbers

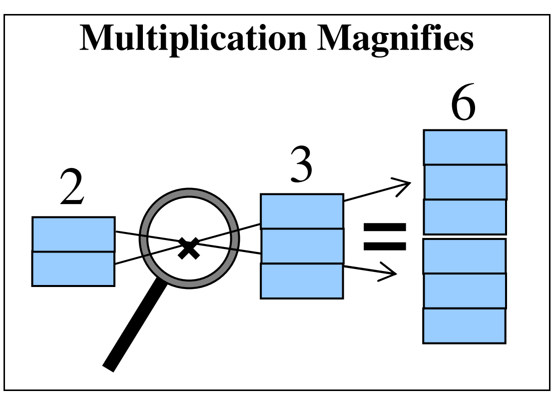
Number: Symbol for a quantity or value.
Natural Numbers: Counting numbers: 1, 2, 3...
Whole Numbers: Zero + Natural numbers: 0, 1, 2, 3...
Integers: Negatives of Natural numbers + Whole numbers: ...-3, -2, -1, 0, 1, 2, 3...

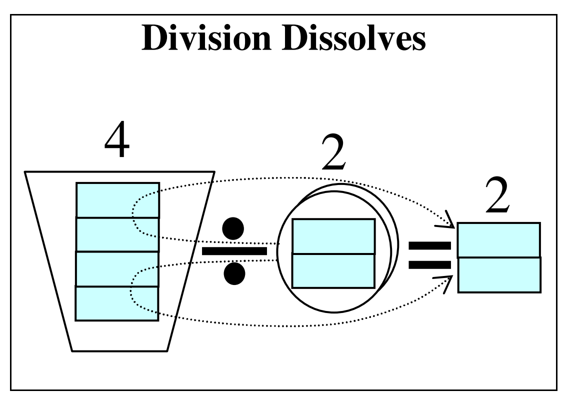
Operators

Operator: Symbol for a procedure (+, –, ×, ÷) or relationship (=, ≠, >, <).







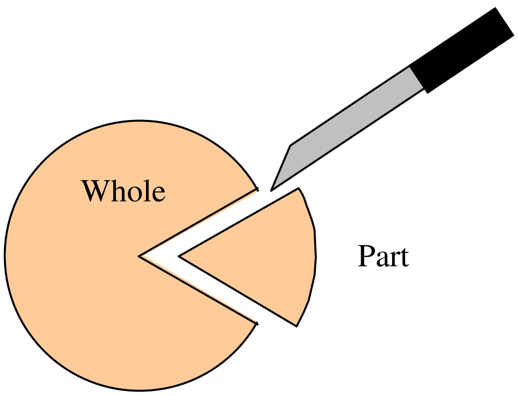


Fraction = POW

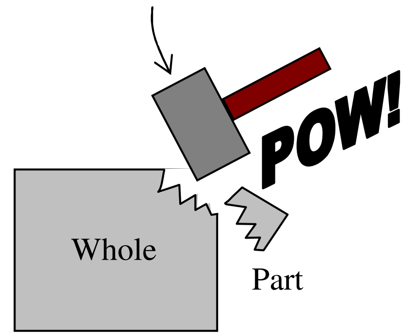
A fraction is a number that is a part of a whole.

BrainAid: Fraction = POW! (Part Of Whole)

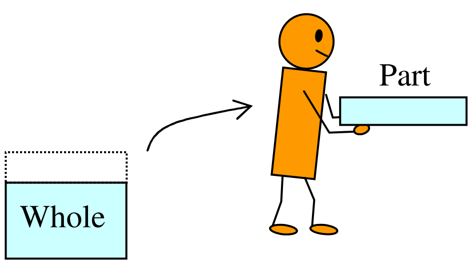
Fraction is from a Latin word that means to fracture or break. Other words that mean *part* include fragment, piece, portion, segment, and subset. Can you think of others?



Imagine slicing a *piece* of a pie.



Imagine breaking off a *fragment* of a concrete block.

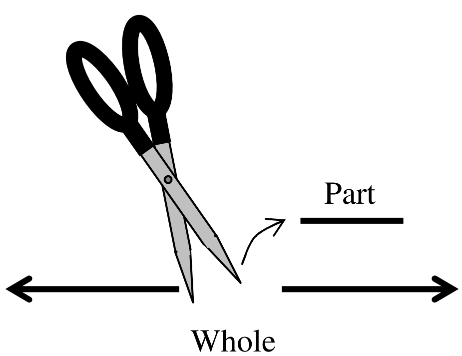


Imagine carrying off a *portion* of a pile.

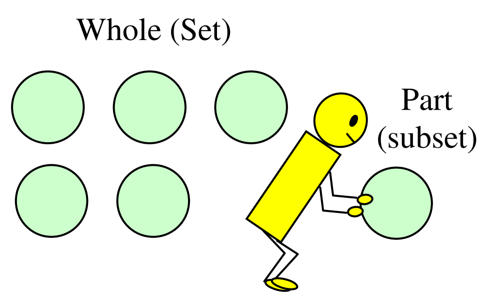
Part/Whole Paradox

Each *part* can be thought of as a new *whole*.

A paradox is a statement that produces an opposite expectation.



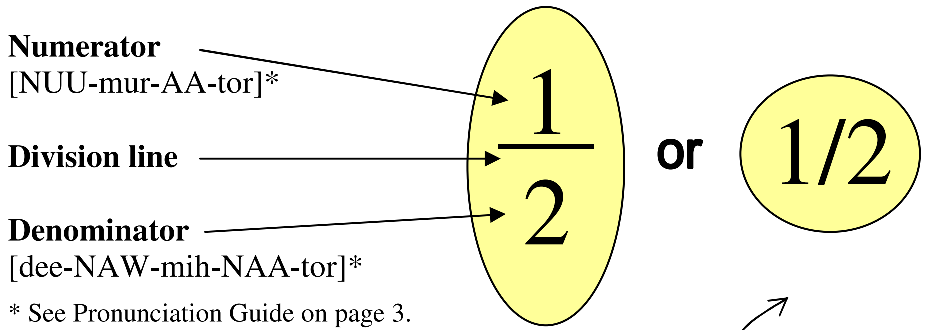
Imagine cutting out a *segment* of a line.



Imagine taking one ball from a set of balls.

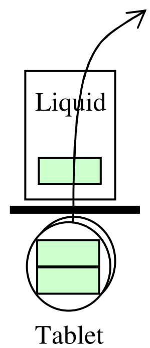
Fraction = Division

A fraction is a division.



BrainAids
Numerator is the number on top.
Denominator is the number down under.

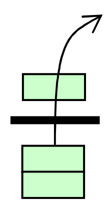
In *Mental Math*, we compared division to a tablet dissolving into a liquid. We'll use the same analogy here.

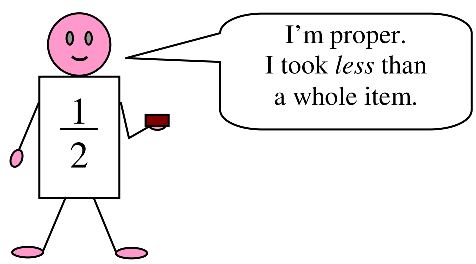


Caution!
The denominator can *not* be zero.
BrainAid
You can't dissolve without a tablet.

Proper Fraction

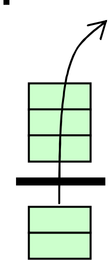
A *proper* fraction is less than 1.
It's what we expect a fraction to be—part of a whole. Its larger denominator will dissolve less than once into its smaller numerator.
Example: 1/2 (one half)

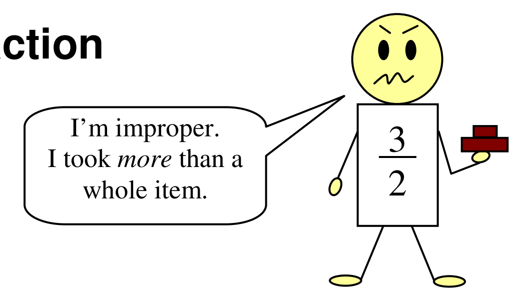




Improper Fraction

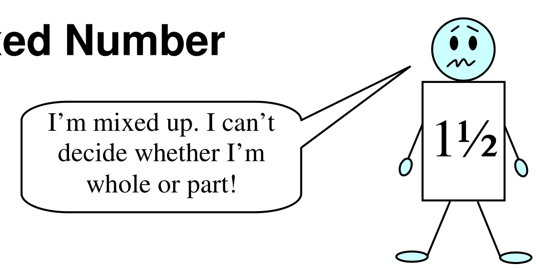
An *improper* fraction is greater than 1.
It's not what we expect from a fraction. Its smaller denominator will dissolve more than once into its larger numerator.
Example: 3/2 (three halves)



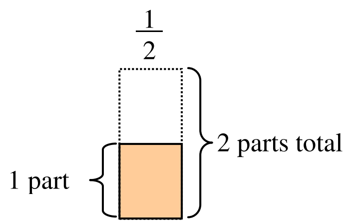
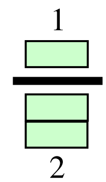


Mixed Number

A mixed number is combination of a whole number and a proper fraction.
Example: 1½ (one and one half)



POW vs. Division View

<table>
<tr><th>POW (Part Of Whole) View</th><th>Division View</th></tr>
<tr><td>We’ll use this view when we want to get a feel for the size of a fraction. A pile represents 1 whole.</td><td>We’ll use this view when we want to manipulate numerators and denominators.</td></tr>
<tr><td>
</td><td>
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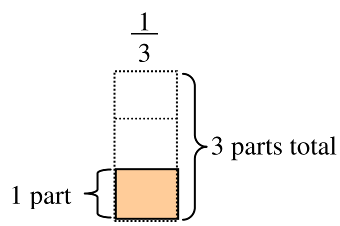
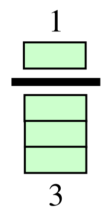
<tr><td>
</td><td>
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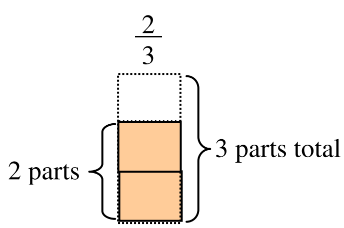
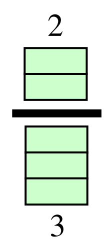
<tr><td>
</td><td>
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Negative Fractions

If either the numerator or denominator are negative, the fraction is negative. If both are negative, the fraction is positive.

In this book, we’ll focus on *positive* fractions. To review the rules for working with negative numbers, please see *Max Learning’s Mental Math*.

$$\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}$$

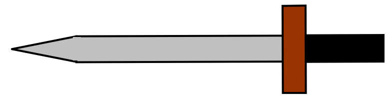
Decimals = Fractions

Decimals are fractions whose denominators are powers of 10.

A decimal [DEH-sih-mul] is a fraction of a whole that has been split into power-of-10 parts, such as 10, 100, 1000, etc.

Decim is Latin for *tenth*. Decimal fractions are written as a decimal point followed by a digit or series of digits.

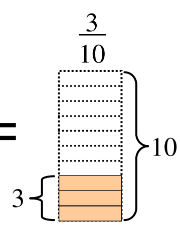
Examples
.3 (point three) = 3/10 (three tenths)
.30 (point three zero) = 30/100 (thirty hundredths)
.03 (point zero three) = 3/100 (three hundredths)
.003 (point zero zero three) = 3/1000 (three thousandths)

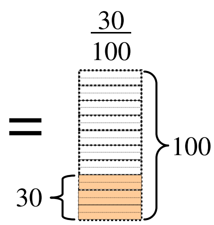


It's True: When the ancient Romans wanted to punish a group of people, they would sometimes *decim*ate (kill) every *tenth* person.

Decimal fractions like .3 are sometimes preceded with a zero as in 0.3 (zero point three) so we don't overlook the decimal point.

An unwritten decimal point is assumed to follow any whole number. For example, 3 is 3. (three point), which is sometimes written as 3.0 (three point zero) so we don't overlook the decimal point.

.3 = 

.30 = 

Mixed Decimals and Place Values

As with a mixed number, a mixed decimal is made from a whole number and a part. Each digit has a place value as shown on the right. The leftmost digits have the greatest value, the rightmost digits have the least value.

Whole | **part**
T h t o • 1/t 1/h 1/T
ones; tens; hundreds; Thousands; tenths; hundredths; Thousandths

Examples
1.7 (one point seven) = 1 7/10 (one and seven tenths)
12.75 (twelve point seven five) = 12 75/100 (twelve and 75 hundredths)
4.235 (four point two three five) = 4 235/1000 (four and two hundred thirty five thousandths)

Why Decimals?

It's usually easier to operate with decimals.	It's usually easier to compare decimals.
$\frac{254}{635} + \frac{76}{152}$ vs. $.4 + .5$ — Which is easier to add?	$\frac{313}{500}$ $\frac{600}{960}$ vs. $.626$ $.625$ — Which makes it easier to find the larger number?

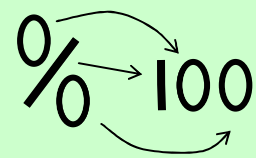
Percents = Fractions

Percents are fractions whose denominators are always 100.

A percent [pur-SENT] is a fraction of a whole that has been split into 100 parts.

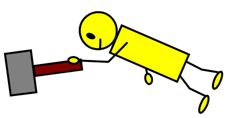
Per centum is Latin for *by the hundred*. *Per* means “divided by” and *cent* means 100, as in 100 cents make one dollar, or a 100 years make one century. Percent fractions are written as a number followed by a percent sign (%).

BrainAid
Visualize the % sign as a 1 with two 0s.

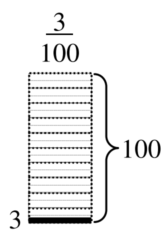


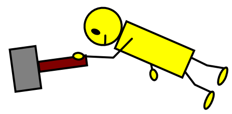
Examples
3% (three percent) = 3/100
3.5% (three point five percent) = 3.5/100
30% (thirty percent) = 30/100
300% (three hundred percent) = 300/100

BrainAid: Pound the % sign down to make it 100.

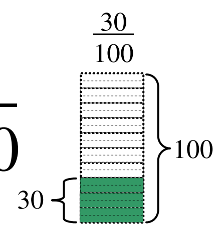


$$3\% = 3\underset{\%}{\downarrow} = \frac{3}{100}$$





$$30\% = 30\underset{\%}{\downarrow} = \frac{30}{100}$$

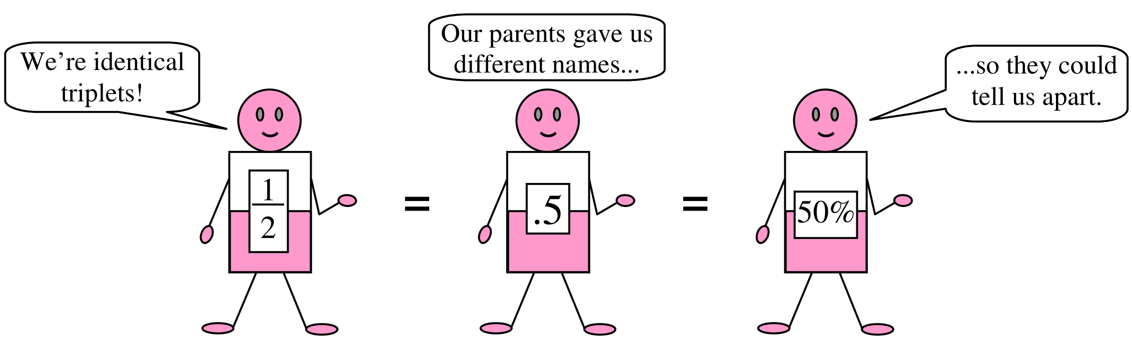


Why Percents?

Percents make *everything* easier to compare.

$\frac{93}{124}$	$\frac{95}{125}$	.75	7.5
↓ 75%	vs. ↓ 76%	↓ 75%	vs. ↓ 750%

Fractions = Decimals = Percents



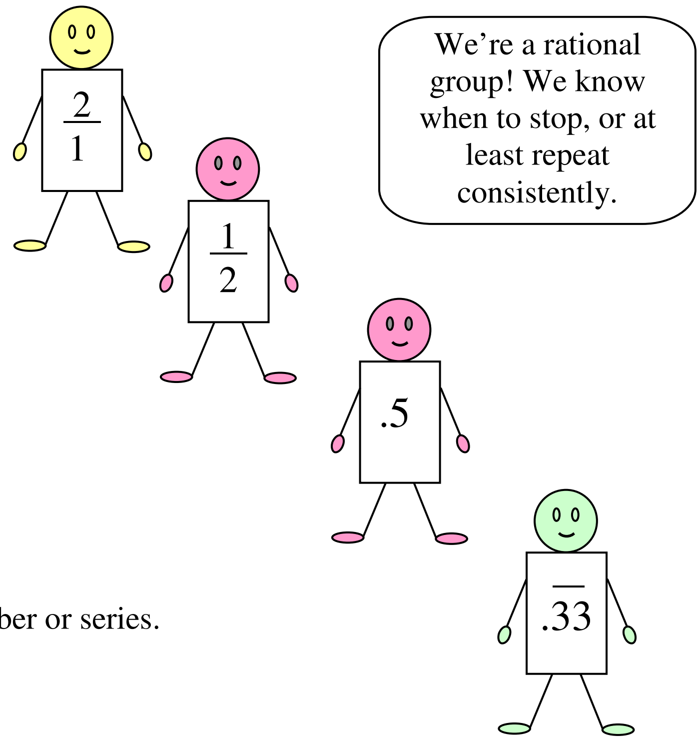
Fraction **Decimal** **Percent**

Rational Numbers

Rational numbers can be written as *ratio*s.

The numbers in a ratio can be separated by a division line (2/1) or a colon (2:1).
BrainAid: A ratio shows the relation between two numbers.

Here are four types of rational numbers:



Integers

All integers can be written over a 1.
Example: 2 can be written as the ratio 2/1.

Fractions

Fractions are already in ratio form.
Examples: 1/2, 5/4.

Terminating Decimals

- Terminate—they come to an end.
- Can be converted to fractions.

Example: 0.5 = 5/10.

Repeating Decimals

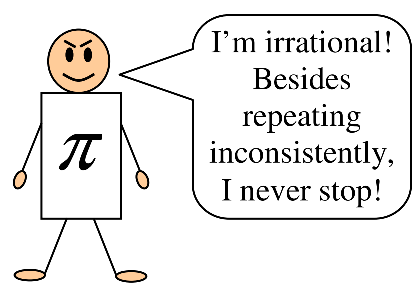
- Nonterminating—they go on forever.
- Repeating—they repeat the same number or series.
- Can be converted to fractions.

Example: 0.33333… = $0.\overline{33}$ = 1/3
The ellipsis (...) means to repeat forever.
The line over .33 means to repeat forever.

BrainAid: Rational people are reasonable. They either stop or repeat consistently.

Irrational Numbers

Irrational numbers can *not* be written as ratios.



- Nonterminating—they go on forever.
- Nonrepeating—they never repeat a series of numbers.
- Can *not* be converted to fractions.

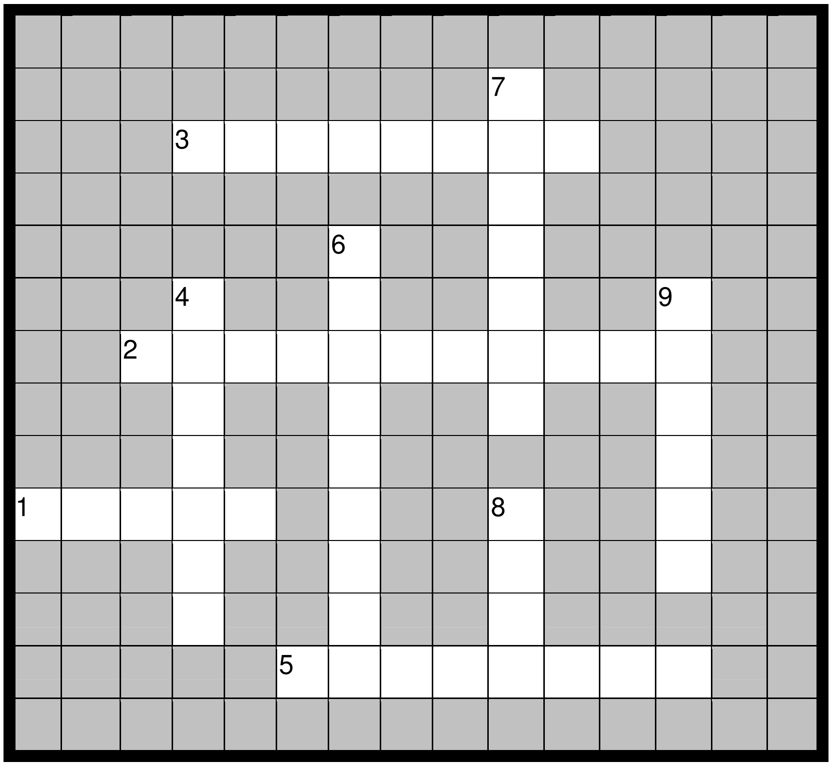
Examples: $\pi = 3.1416...$ $\sqrt{3} = 1.73205...$

BrainAid: Irrational people are unreasonable.
They go on and on without rhyme or reason.

Real Numbers

Rational and Irrational Numbers make up the Real Number system.
It sounds strange, but there is also an *Imaginary* Number System.

BrainDrain #1



Fill in the Crossword Puzzle

Across

1. A _____ number has a whole and a part.
2. The _____ is the bottom part of a fraction.
3. This type fraction is greater than 1.
5. A _____ is also a division.

Down

4. _____ fractions have denominators of 100.
6. The _____ is the top part of a fraction.
7. _____ fractions have power-of-10 denominators.
8. A fraction is a _____ of a whole.
9. A _____ fraction is less than 1.

True/False

Write T or F in the blanks.

1____ Rational numbers are ratios.

2____ Integers are rational.

3____ Fractions are rational.

4____ Rational numbers can repeat forever.

5____ Irrational numbers can't be ratios.

6____ Irrational numbers never end.

Study Buddy

If you struggle with math in school, find a study buddy to compare answers with until your ability and confidence grow. In exchange for math help, offer to help your study buddy in other ways.

Fractions In The Newspaper

The most common place to see fractions, usually in the form of decimals or percents, is in the newspaper, especially the business section. It seems there's always something that's a part of a whole, or going up or down a certain percent. See how many examples you can find in today's paper.

Operations

Equivalent Fractions

Equivalent Fractions are equal in value but not in appearance.

Fundamental Property of Fractions
Multiplying or dividing *both* the numerator and denominator by the *same* number creates an equivalent fraction.

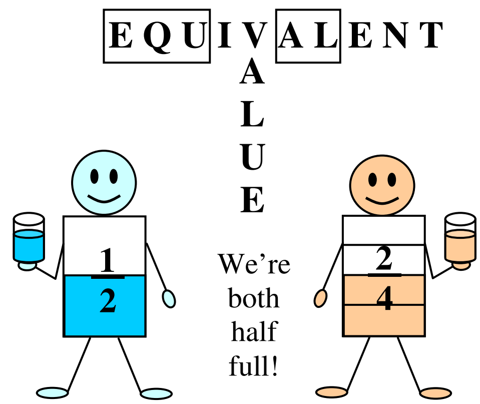
– or –

Fun Property of Fractions
Both top and bottom wanna be hoppin!
Terms just wanna have fun!

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Equals 1

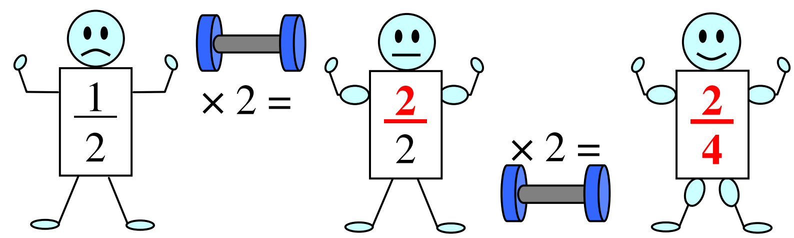
$$\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$



Why it works.
Multiplying or dividing a number by 1 does not change its value.
e.g.; $5 \times 1 = 5$

Multiplying or dividing a fraction by the fractional *equivalent* of 1 changes the fraction's *appearance* but not its value.

Multiply Muscle Builds



This MathBot is unhappy about his puny muscles.

Arm weights *multiply* his upper *muscles,* but now he's top heavy.

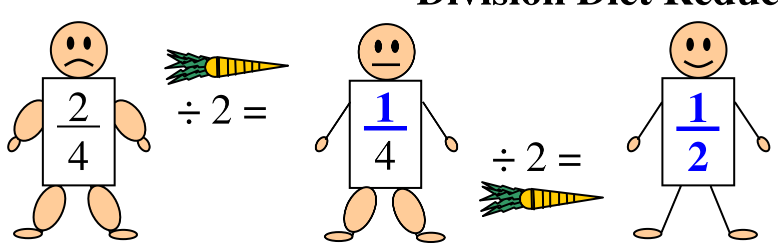
Leg weights *multiply* his lower *muscles*. Now he's buff all over and having fun!

TRAP!
This property does *not* work with addition or subtraction.

$$\frac{1+2}{2+2} \neq \frac{3}{4}$$

$$\frac{3-2}{4-2} \neq \frac{1}{2}$$

Division Diet Reduces



This MathBot is unhappy about his weight.

A *division diet* and upper-body aerobics reduce his upper body, but he's still bottom heavy.

A *division diet* and lower-body aerobics do the trick. Now he's thin and trim all over and having fun!

Making Multiples

Before making equivalent fractions, let's review the
concept of multiples that was introduced in *Mental Math*.

Multiples are *products* created by multiplying a base number times a series of numbers.

Base × Number = Multiple

Common Multiples are multiples that are the same for different bases.
In the table, you can see that 6 and 12 are common multiples of the bases 2 and 3.

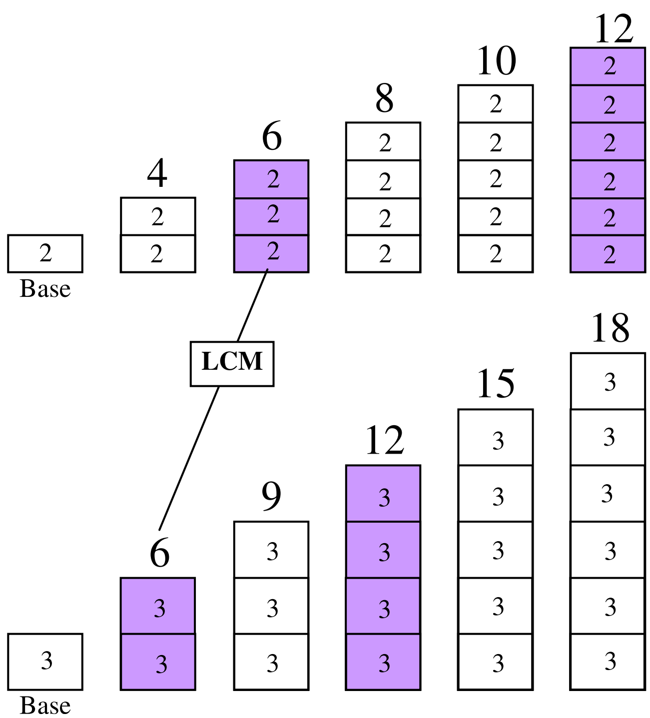
Number Series

	×	2	3	4	5	6	
Base	**2**	4	6	8	10	12	**Multiples of 2**
Base	**3**	6	9	12	15	18	**Multiples of 3**

LCM

The **LCM** is the Least Common Multiple (i.e., the smallest).
Observe that 6 is the LCM of the bases 2 and 3.

BrainAid: Think of <u>m</u>ultiples as a series of <u>m</u>ounds (aka piles) built from a base.
For example, the base 2 mounds below are 4, 6, 8, 10, and 12. And we could keep on building.



BrainAid

<u>M</u>ultiples are <u>M</u>ore than the numbers that produce them.

(As long as the numbers are positive and greater than 1.)

Your turn: Fill in the following Multiples Table. Circle common multiples.

×	2	3	4	5	6
4	8				24
5	10				30

Making Equivalent Fractions

We use the same Base × Number = Multiple formula to create equivalent fractions, except each Base is a fraction and each Number is a fraction that is equivalent to 1. This creates two series of multiples: one for the numerator and one for the denominator.

Common Denominators are denominators that are the same for different fractions. In the table, observe that 6 and 12 are common denominators.

Fraction Series (all are equal to 1)

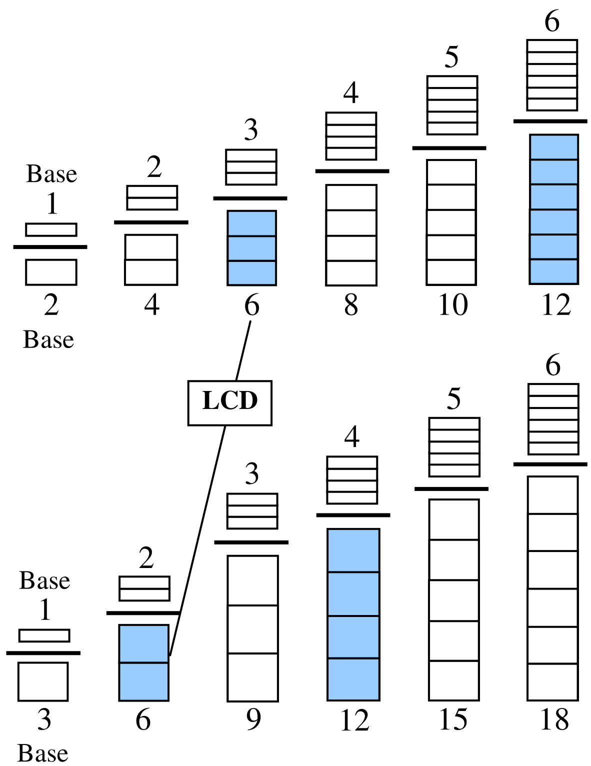
	×	2/2	3/3	4/4	5/5	6/6	
Base	**1/2**	2/4	3/6	4/8	5/10	6/12	**Equivalents of 1/2**
Base	**1/3**	2/6	3/9	4/12	5/15	6/18	**Equivalents of 1/3**

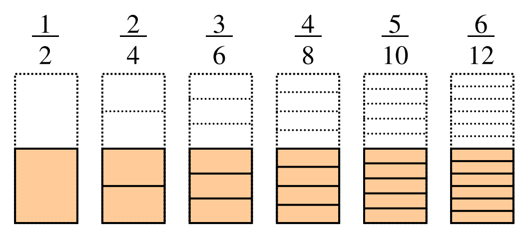
LCD

The LCD is the LCM of the denominators.

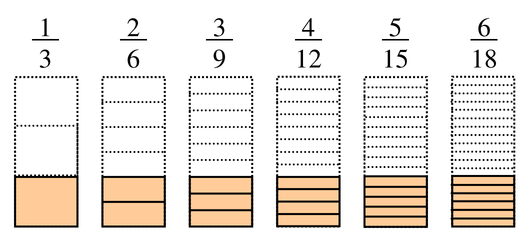
The **LCD** is the Least Common Denominator (i.e., the smallest). Observe that 6 is the LCD of the equivalents for 1/2 and 1/3.

For each equivalent fraction, the proportion of numerator to denominator remains the same.	Each equivalent fraction represents the same part of a whole.





These are all equivalent to 1/2



These are all equivalent to 1/3

Your turn: Fill in the following Equivalent Fractions Table. Circle common denominators.

×	2/2	3/3	4/4	5/5	6/6
2/3	4/6				12/18
3/4	6/8				18/24

Reduced Fractions

A reduced fraction is an *equivalent* fraction with a *smaller* numerator and denominator.

Why Reduce?

It's easier to operate with reduced fractions.	It's easier to visualize the size of a reduced fraction.
Original $\frac{48}{96} \times \frac{17}{51}$ $\downarrow$ vs. $\downarrow$ Reduced $\frac{1}{2} \times \frac{1}{3}$ Which is easier to multiply?	Original $\frac{368}{736}$ vs. Reduced $\frac{1}{2}$ Which is easier to visualize?

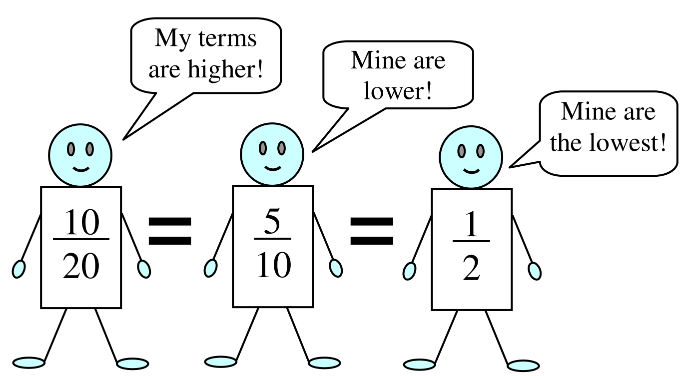
Lowest Terms

Because smaller numbers are easier to work with, it's usually to our advantage to reduce a fraction to its lowest terms (aka simplest form).

In this case, the word *terms* refers to the numerator and denominator.

The goal is to reduce until we reach the *smallest* possible numerator and denominator.

The following fractions are equivalent, but only the last is in lowest terms.



Larger vs. Smaller Equivalents

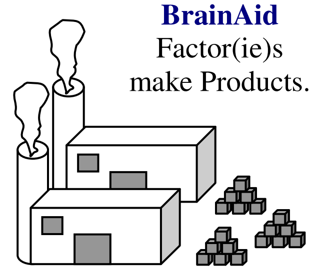
Multiplying (making multiples) creates larger equivalent fractions.

Dividing (reducing) creates smaller equivalent fractions.

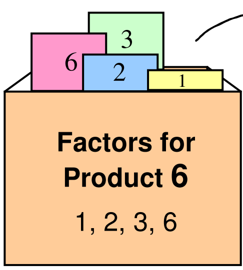
Finding All Factors

Before learning to reduce fractions, let's review the concept of factors first introduced in *Max Learning's Mental Math*.

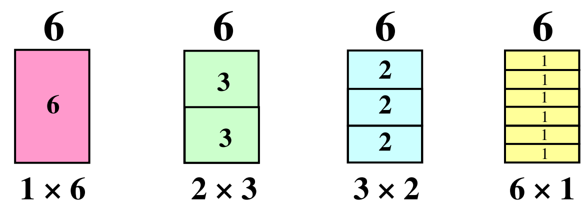
Factors = Multipliers
Factor × Factor = Product
Factoring = Extracting Multipliers



BrainAid
Factor(ie)s make Products.



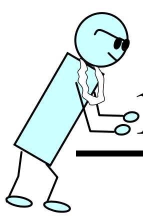
BrainAid
All the factors that make a product are kept in a box waiting for assembly.



Product 6 can be 'assembled' using different factors.

All-In-The-Box Factoring

To find *all* pairs of factors of a product, use the *Factor Extractor* Box!



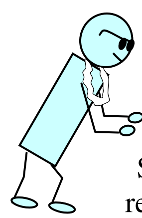
12

Place product to be factored on top of the box 'lid.'

12
1 × 12

12
1 × 12
2 × 6

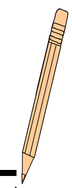
Factor sequentially in 1, 2, 3 order. Skip factors that don't divide *exactly* (without remainder).



12
1 × 12
2 × 6
3 × 4
~~4 × 3~~

Stop at first reversed pair.

12
1 × 12
2 × 6
3 × 4



Draw arrow to complete box.

Common Factors & Greatest Common Factor (GCF)

Factors that are the same for different products are called *common* factors. To demonstrate, we'll factor 20, arrange the factors of 12 and 20 in order, then circle all the common factors.

20
1 × 20
2 × 10
4 × 5

Product	Factors					
12	1	2	3	4	6	12
20	1	2	4	5	10	20

GCF

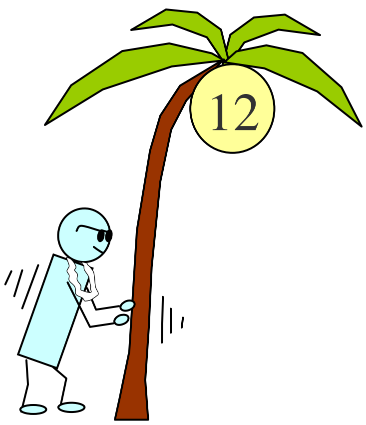
Observe that the common factors of 12 and 20 are 1, 2, and 4.

The **GCF** of 12 and 20 is 4,
which is the "greatest" of the common factors.

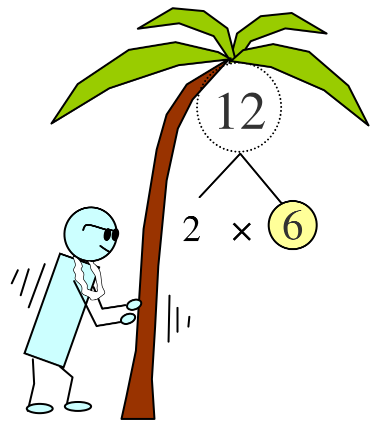
GCF Paradox
The *Greatest* Common Factor is *less* than the products it's derived from.

Finding Prime Factors

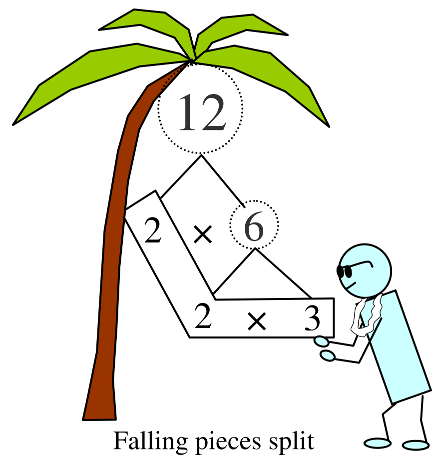
Prime numbers are exactly divisible by one and themselves only.



Factor Freddie is hungry. So he grips and shakes a palm tree to dislodge the *product* of his labor: a ripe coconut filled with nutrients.



He shakes so hard that as it falls the coconut splits into smaller pieces (*factors*).



Falling pieces split into their *prime* nutrients, and Freddie collects them in his lunch basket.

Factoring Tricks from *Max Learning's Mental Math* reveal if a product is divisible by a particular factor without having to first divide it. If you can't remember the tricks, or don't have them handy, use trial and error to divide out prime factors in ascending order—2, then 3, then 5, then 7, then 11, etc.

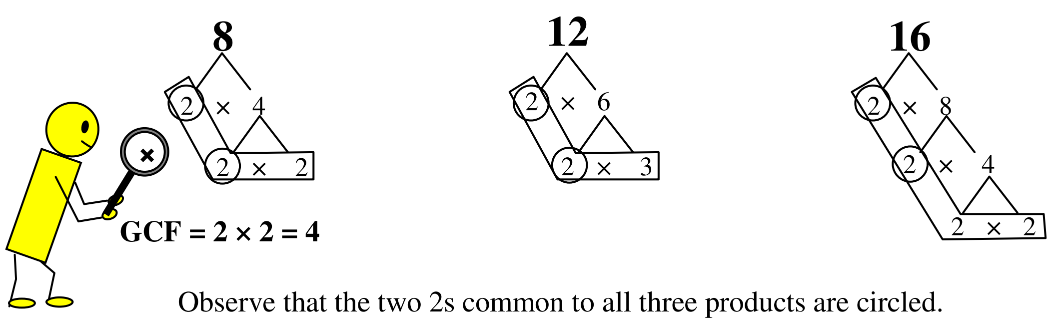
Finding the GCF

Grip, Catch, Focus

To find the GCF of several products:

- Grip each products' Factor Tree, and shake out its prime factors.
- Catch (circle) factors that are common to all products each time they occur.
- Focus on and magnify (multiply) any *one* set of circled factors to get the GCF.

Example: Find the GCF of 8, 12, and 16.



Observe that the two 2s common to all three products are circled.
Observe that only *one* set of common factors is multiplied to find the GCF.

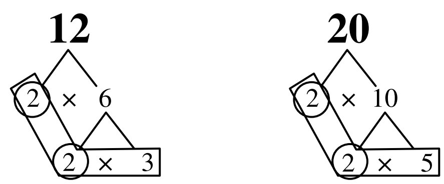
Your turn: Find the prime factors and GCF of each pair of products.

12 **27**	**30** **50**	**22** **35**
GCF = ________________	**GCF =** ________________	**GCF =** ________________

Reduce with GCF

Problem: Reduce 12/20 to its lowest terms.

Solution: Find the GCF of 12 and 20 and use it to reduce each term.



GCF = 2 × 2 = 4

BrainAid
Imagine dissolving a tablet into each term.

Equals 1

$$\frac{12}{20} \div \frac{4}{4} = \frac{3}{5}$$

Why it works: Dissolving the numerator and denominator equally creates a smaller equivalent fraction. Dissolving by the GCF ensures the lowest terms since the remaining numbers have no common factors.

TRAP!

If you don't *completely* factor to primes, you may quit factoring too soon and not reach lowest terms.

Example: Is 17/51 in lowest terms? No, because 51 = 3 x 17.

It may take longer to factor to primes, but it's better to be slow and right than fast and wrong!

Your turn: Find the GCF and reduce each fraction to it lowest terms.

12 **15**	**16** **20**	**18** **27**
$\frac{12}{15} \div \frac{\bigcirc}{\bigcirc} =$	$\frac{16}{20} \div \frac{\bigcirc}{\bigcirc} =$	$\frac{18}{27} \div \frac{\bigcirc}{\bigcirc} =$

Reduce with Numerator

Problem: Reduce 3/12 to its lowest terms.

Solution: You observe that the numerator is a factor of the denominator, so you dissolve the 3 directly into the 12. This is a variation of the GCF method except that here the numerator *is* the GCF.

BrainAid

As a tablet dissolves, it leaves a residue of 1 behind.

An arrow *through* the reduced terms ensures you don't use them again.

Residue

$$\frac{\overset{1}{3}}{\underset{4}{12}} = \frac{1}{4}$$

Equals 1

$$\frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

Your turn: Reduce each fraction to its lowest terms by dissolving the numerator into the denominator.

$\frac{3}{15} =$	$\frac{4}{16} =$	$\frac{5}{35} =$

Now you draw the dissolving tablets and arrows, then reduce.

$\frac{6}{36} =$	$\frac{7}{21} =$	$\frac{8}{80} =$

Reducing Improper Fractions

With an improper fraction, if the denominator is a factor of the numerator, this method also works. However, the end result is a whole number, so it's more of a division problem than a reducing problem.

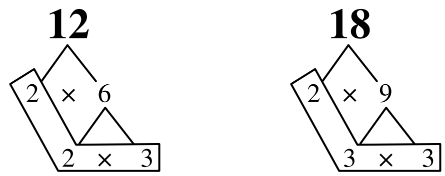
Example: Reduce 12/3 to its lowest terms.

$$\frac{\overset{4}{12}}{\underset{1}{3}} = \frac{4}{1} = 4$$

Reduce with Prime Factors

Problem: Reduce 12/18 to its lowest terms.

Solution: You factor both the numerator and denominator into their prime factors, dissolve common factors, then multiply remaining factors. This is a more direct way to reduce because it eliminates the need to calculate then divide by the GCF.



BrainAid
Each tablet dissolves into its counterpart 1 time and leaves a residue of 1 behind.

$$\frac{12}{18} = \frac{\overset{1}{2} \times 2 \times \overset{1}{3}}{\underset{1}{2} \times \underset{1}{3} \times 3} = \frac{2}{3}$$

You can dissolve up or down, vertically or diagonally as you choose.

Canceling?
You may have heard this technique called “canceling like terms.” But canceling implies eliminating, which implies that the numbers disappear to zero. In reality, each “canceled” term becomes a 1.

TIP!
Take the time to factor completely to primes. Remember, it doesn’t matter how quickly you solve a problem if you do it wrong.
Do It Right The First Time!
(DIRT FooT!)

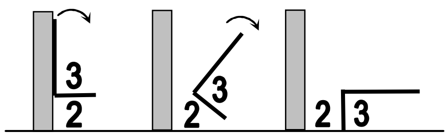
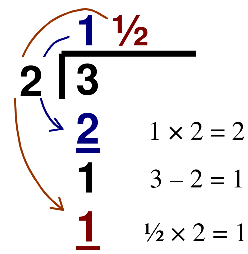
Your turn: Factor to primes, then reduce each fraction to its lowest terms.

14 **20**	**16** **24**
$\frac{14}{20}$ = ________ =	$\frac{16}{24}$ = ________ =

Converting Improper & Mixed

Divide Improper to Get Mixed

To convert an improper fraction (greater than 1)
to a mixed number (whole + fraction),
divide the numerator by the denominator.

<table>
<tr><th>In Mental Math, we converted a fractional division to long division by imagining a table being unbolted from a wall and rotated to the floor.</th><th>We called it Rainbow Division because the motion of long division resembles the arcs of a rainbow. The result is a mixed number.</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Your turn: Divide the improper fractions to get mixed numbers.

$\frac{4}{3} =$	$\frac{5}{3} =$	$\frac{7}{3} =$
$\frac{6}{4} =$ Tip: ½ × 4 = 2	$\frac{6}{5} =$	$\frac{11}{3} =$

Add Mixed to Get Improper

Adding the whole number and fraction of a mixed number creates an improper fraction.
Although an addition problem, it's easier to use this traditional conversion trick.

Mixed Number	Multiply denominator times whole number.	Add product to numerator.	Place sum over denominator to make an improper fraction.
$1\frac{1}{2}$	2 $1\frac{1}{2}$	2 + $1\frac{1}{2}$	$\frac{3}{2}$

Here's how it looks in condensed form.

2 +

$1\frac{1}{2} = \frac{3}{2}$

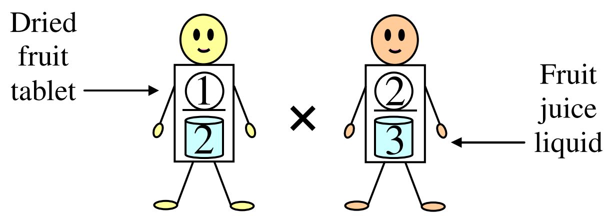
Why it works: See *Mixed To Improper Half Spotlighting* on page 37.

Your turn: Use the traditional conversion trick to convert the mixed numbers to improper fractions.

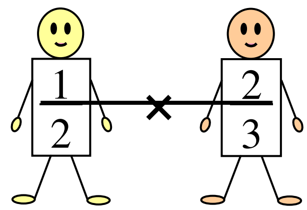
$2\frac{1}{5} =$	$2\frac{3}{4} =$	$3\frac{1}{3} =$
$3\frac{4}{6} =$ Tip: First reduce the 4/6.	$4\frac{1}{3} =$	$4\frac{2}{7} =$

Multiplying Fractions:
Merge, Melt, & Magnify

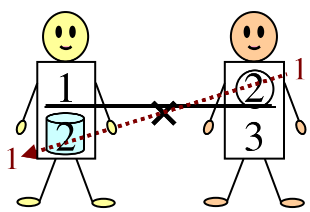
BrainAid: Imagine MathBots have eaten dried fruit (tablets) and juices (liquids) that can be dissolved across digestive membranes, then magnified across top and bottom.



Merge fraction bars to make one long membrane.

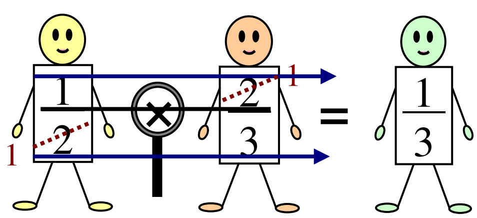


Melt Dissolve where possible across the membrane.



Tablets dissolving into liquids leave a powdery residue of '1' behind.

Magnify across top and bottom.



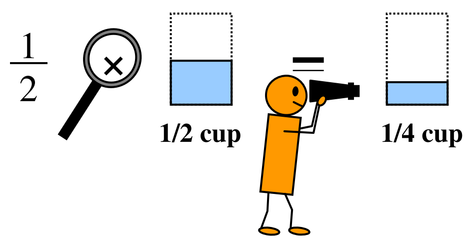
Compact Version

$$\frac{\overset{}{1}}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{3} = \frac{1}{3}$$

Paradox

Magnifying proper fractions creates a *smaller* fraction.

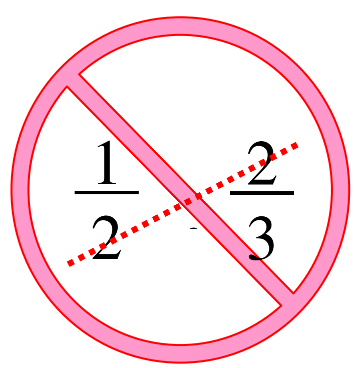
It creates a *smaller* fraction because you're taking "part of a part."



BrainAid

Imagine looking backwards through binoculars.

Trap! Do *not* cross melt when adding! There is no membrane to melt across!



Proper Multipliers = Smaller Product

Your turn: Merge, melt (if possible), and magnify.

$\frac{1}{3}\times\frac{1}{4}=$	$\frac{2}{3}\times\frac{2}{5}=$	$\frac{2}{3}\times\frac{5}{7}=$

Melt Before Magnifying

Question: Why merge the fraction bars?

Answer: To create a "membrane" through which terms can be melted (aka dissolved, divided, reduced), *across* fractions. Cross-reducing creates smaller terms that are easier to magnify and result in a product in lowest terms.

A membrane is a thin layer of material, like your skin, that things can dissolve through.

Numerator Cross-Reduce	GCF Cross-Reduce
$\frac{\overset{1}{4}}{5}\times\frac{3}{\underset{2}{8}}=\frac{3}{10}$	$\frac{\overset{2}{6}}{7}\times\frac{5}{\underset{3}{9}}=\frac{10}{21}$
Numerator Reduce, then Cross-Reduce	**Prime Factor Cross-Reduce**
$\frac{\overset{1}{2}}{\underset{\underset{1}{3}}{6}}\times\frac{\overset{3}{9}}{11}=\frac{3}{11}$	$\frac{\overset{\overset{1}{2}\times\overset{1}{2}}{4}}{\underset{3\times\underset{1}{3}}{9}}\times\frac{\overset{\overset{1}{2}\times\overset{1}{3}}{6}}{\underset{\underset{1}{2}\times\underset{1}{2}\times\underset{1}{2}}{8}}=\frac{1}{3}$

Your turn: Merge, melt, and magnify the following sets of fractions.

$\frac{2}{7}\times\frac{3}{4}=$	$\frac{2}{4}\times\frac{3}{9}=$	$\frac{8}{14}\times\frac{7}{12}=$
$\frac{8}{9}\times\frac{6}{10}=$	$\frac{4}{21}\times\frac{14}{16}=$	$\frac{9}{18}\times\frac{6}{15}=$

Improper Multiplier = Larger Product

Since improper fractions are greater than 1, their product is larger than either fraction.
This is easier to see when the improper fractions are converted to mixed numbers below.

$$\frac{3}{2} \times \frac{9}{4} = \frac{27}{8}$$

$$1\frac{1}{2} \qquad 2\frac{1}{4} \qquad 3\frac{3}{8}$$ is larger than either multiplier.

If one of the multipliers is a whole number, you can multiply across the top...	...or you can rewrite the whole number as an improper fraction with a denominator of 1.
$2 \times \frac{4}{3} = \frac{8}{3}$	$\frac{2}{1} \times \frac{4}{3} = \frac{8}{3}$

Your turn: Multiply the following improper fractions, then convert all to mixed numbers.

$\frac{3}{2} \times \frac{5}{4} =$	$\frac{4}{3} \times \frac{5}{3} =$	$\frac{8}{5} \times \frac{4}{3} =$

Proper × Improper = In-Between Product

The product will be larger than the proper fraction (since you're magnifying it by more than 1) and smaller than the improper fraction (since you're magnifying it by less than 1). Therefore, the product will be in between the two fractions. Example: 1/3 × 3/2 = 1/2 (which is in between 1/3 and 3/2).

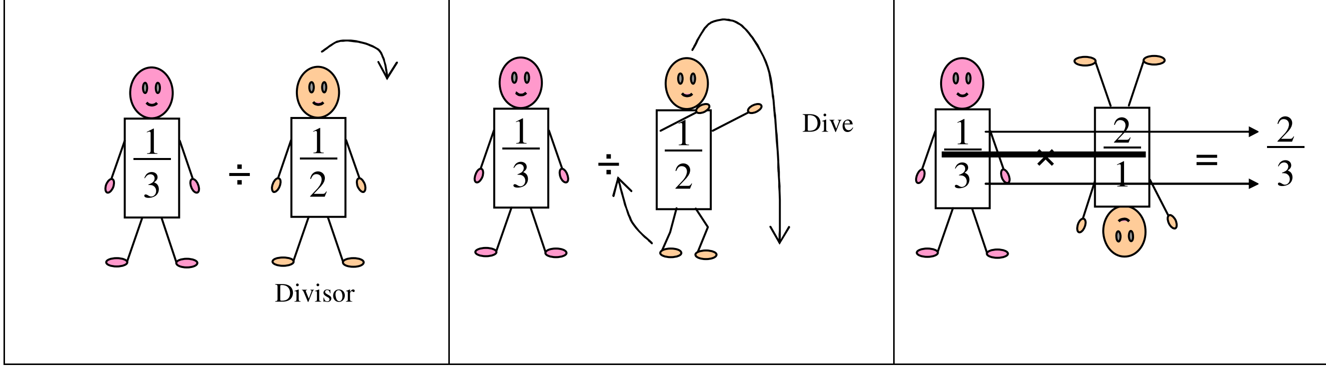
Your turn: Multiply the following proper & improper fractions, then convert improper to mixed.

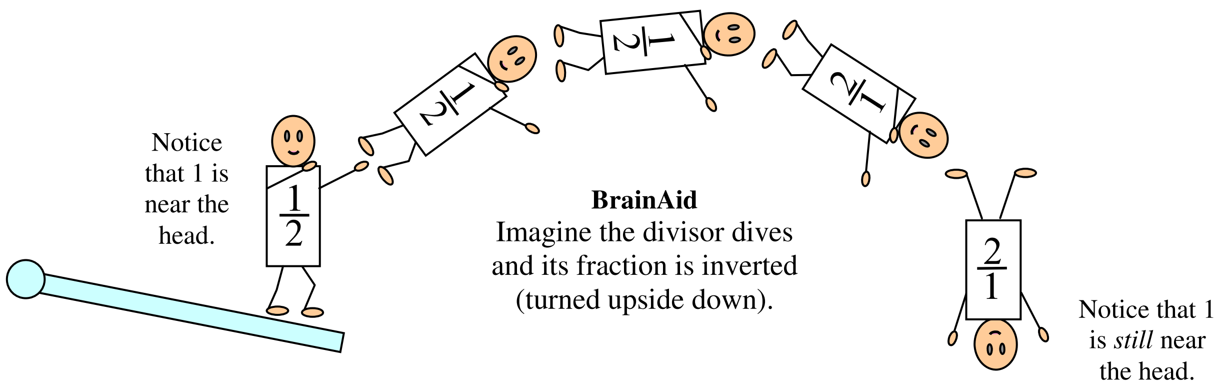
$\frac{5}{3} \times \frac{1}{4} =$	$\frac{7}{5} \times \frac{1}{2} =$	$\frac{2}{3} \times \frac{4}{3} =$

Dividing Fractions:
Flip & Multiply

Dive the Divisor

For horizontal layouts, dive the divisor.	As it flips, its heel kicks the ÷ changing it into a ×.	Now multiply with merge, melt, & magnify.





BrainAid
Imagine the divisor dives and its fraction is inverted (turned upside down).

Your turn: Dive the divisor and multiply (merge, melt, magnify) the following fractions.

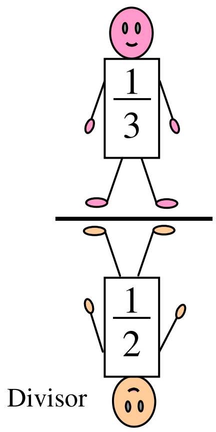
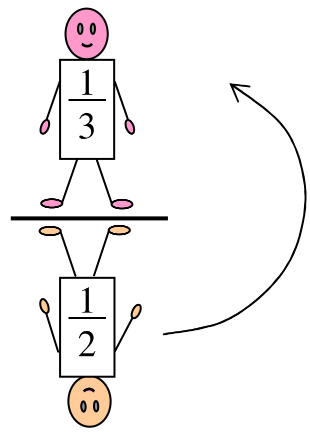
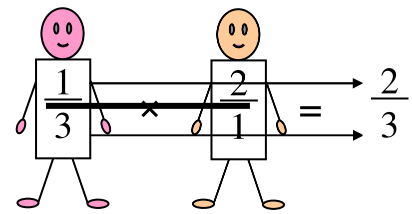
$$\frac{2}{3} \div \frac{3}{4} \longrightarrow \frac{2}{3} \times \frac{}{} =$$

$$\frac{2}{3} \div \frac{2}{5} \longrightarrow \frac{2}{3} \times \frac{}{} =$$

$$\frac{5}{8} \div \frac{5}{4} \longrightarrow \frac{5}{8} \times \frac{}{} =$$

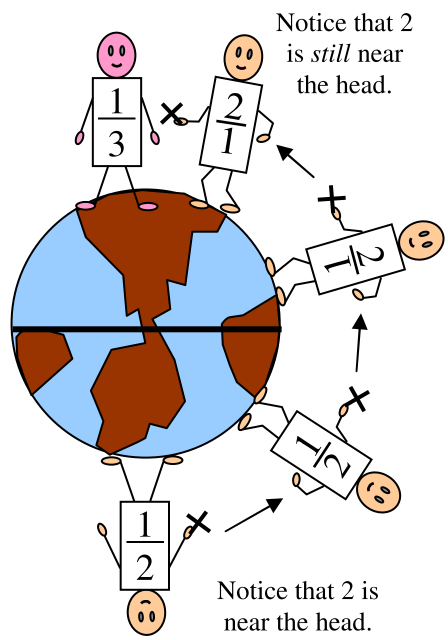
Divisor Down Under

A fraction vertically divided by another fraction is called a *Complex Fraction*.
But there's nothing really complex about it.

For vertical layouts...	...flip the divisor to the top...	...then multiply.
		

BrainAid
The Divisor Down Under, carrying a replica of the Southern Cross* as a gift, walks around the globe, crossing the equator to visit a northern friend.

* The Southern Cross is a star constellation visible from the southern hemisphere.



Your turn: Flip the divisor and multiply the following fractions.

$\dfrac{\frac{2}{3}}{\frac{4}{3}} \longrightarrow \frac{2}{3} \times \frac{\quad}{\quad} =$	$\dfrac{\frac{5}{8}}{\frac{2}{5}} \longrightarrow \frac{5}{8} \times \frac{\quad}{\quad} =$

Fractional Division Issues

Question: When dividing fractions, why must I flip the divisor?

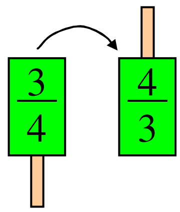
Answer: One goal of division is to get a “1 on the bottom,” i.e., make the divisor equal to 1 so no more division is needed. To do so, we multiply the divisor fraction by its reciprocal. Then to maintain equivalency, we multiply the dividend fraction by the same reciprocal. So in effect, we are multiplying the overall division by 1, which doesn’t change its value.

Inverse = Reciprocal

A flipped fraction is called an inverse or reciprocal [ree-SI-proh-kul].

BrainAid

Imagine a reciprocal is a popsicle that flipped over. Think re*flip*rocal!



Dividend Fraction

Divisor Fraction

$$\frac{\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}}{\frac{1}{2} \times \frac{2}{1} = \frac{2}{2}} = \frac{\frac{2}{3}}{1} = \frac{2}{3}$$

Equals 1

Reciprocal

Equals 1

“1 on the bottom”

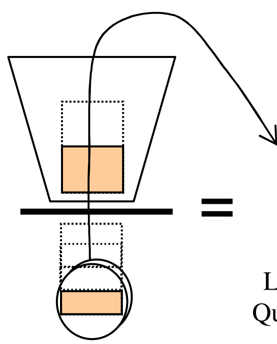
Question: Normally when I divide, the quotient is smaller than the dividend. How come when I divide by a fraction, the quotient is *larger* than the dividend?

Answer: When the divisor is a *proper* fraction (i.e., a part), the quotient will be larger because a ‘part’ will fit more times into a dividend than a ‘whole’ would. On the other hand, if you’re dividing by an *improper* fraction, which is more than a whole, the quotient will be *smaller* than the dividend.

Dividend

Proper Divisor

$$\frac{\frac{1}{2}}{\frac{1}{4}} =$$



$$= 2$$

Larger Quotient

The smaller 1/4 dissolves into the larger 1/2 *two* times.

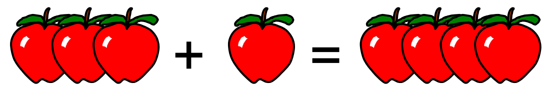
Your turn: Divide, then compare quotients for the following proper and improper divisors.

$\frac{\frac{3}{5}}{\frac{1}{5}} =$	$\frac{\frac{3}{5}}{\frac{5}{1}} =$

Adding Fractions

Adding Like Fractions

'Like' fractions have identical denominators.

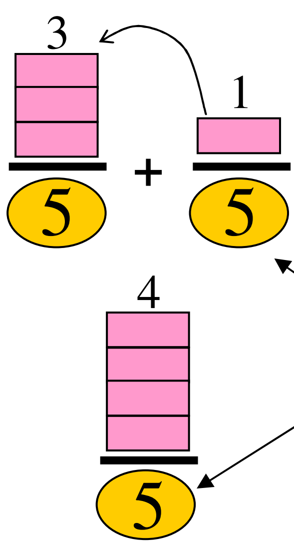


3 apples + 1 apple = 4 apples

It's easy to add apples.

If you had a different kind of fruit called "fifths," they'd be just as easy to add.

3 fifths + 1 fifth = 4 fifths



***Fifth* fruit**

BrainAid

Imagine denominators are fruit. 'Like' fractions have the *same* fruit on the bottom.

To add 'like' fractions, attach the numerators over one denominator.

Trap! Do *not* add denominators! You'd change the type of fruit!

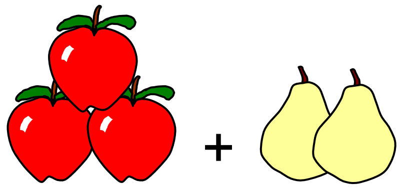
$$\frac{3}{5} + \frac{1}{5} \neq \frac{4}{10}$$

Your turn: Draw your choice of fruit shape around like denominators. Add. Reduce if needed.

$\frac{1}{4} + \frac{2}{4} =$	$\frac{3}{5} + \frac{2}{5} =$	$\frac{4}{7} + \frac{2}{7} =$
$\frac{1}{8} + \frac{3}{8} =$	$\frac{2}{6} + \frac{4}{6} =$	$\frac{7}{2} + \frac{4}{2} =$

Adding Unlike Fractions

"Unlike" fractions have different denominators.

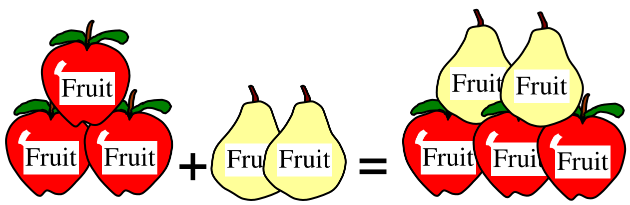


3 apples + 2 pears = ???

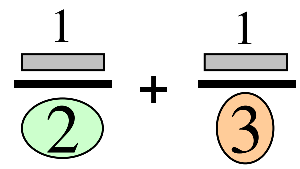
Problem: Can't meaningfully add unlike items like apples and pears.

Question: Do they have anything in common?

Answer: They are all fruits!



3 fruits + 2 fruits = 5 fruits

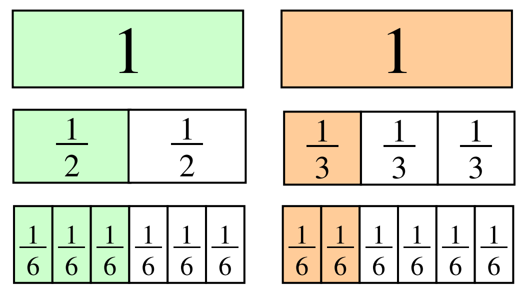


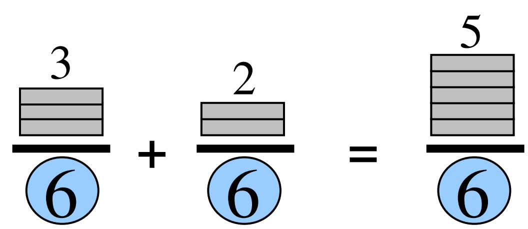
1 half + 1 third = ???

Problem: Can't meaningfully add unlike fractions like halves and thirds.

Question: Do they have anything in common?

Answer: They can both be split into sixths!





3 sixths + 2 sixths = 5 sixths

Question: **How do we make unlike fractions into like fractions?**

Answer: **Create equivalent fractions with common denominators!**

LCD Paradox
The *least* common denominator is *greater* than the original denominators.

Equivalent Fraction Table

bases	×	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	**Fraction Series** (all equal 1)
	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	Equivalents of 1/2
	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$	Equivalents of 1/3

Common Denominators: 6, 12

LCD = Least Common Denominator = 6

The LCD is the *smallest* multiple (aka product) that the original denominators will dissolve into.

Using the LCD keeps the equivalent fractions small and easy to work with and the answer at or near lowest terms.

Spotlighting

Spotlighting is a cool algorithm for creating equivalent fractions with common denominators.

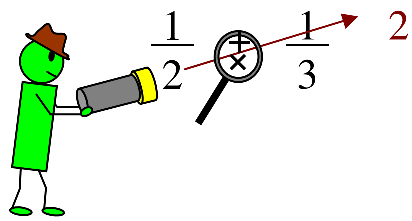
Case 1: Denominators With No Common Factors

N = Numerator
D = Denominator

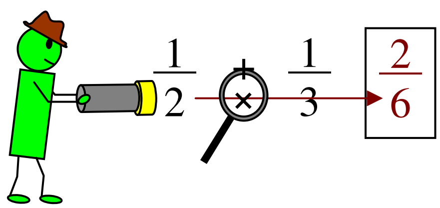
$$\frac{1}{2} + \frac{1}{3}$$

Paradox
To *add* unlike fractions, you must first *multiply*.

Multiply left D times right N.

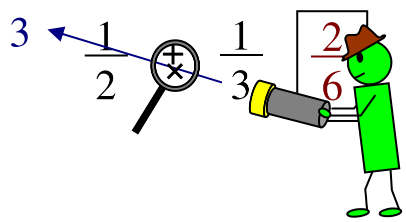


Multiply Ds across bottom.

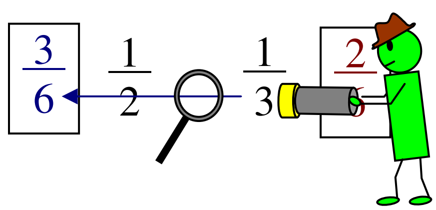


Draw box around right equivalent fraction.

Multiply right D times left N.



Draw box around left equivalent fraction.



Multiply Ds across bottom.

Attach boxed numerators.

$$\boxed{\frac{3}{6}}\ \frac{1}{2} + \frac{1}{3}\ \boxed{\frac{2}{6}} = \mathbf{\frac{5}{6}}$$

Circle the answer.

Your turn: Spotlight to add unlike fractions with *no* common factors in denominators.

$\boxed{}\ \frac{1}{2} + \frac{2}{3}\ \boxed{} =$	$\boxed{}\ \frac{1}{3} + \frac{1}{4}\ \boxed{} =$
$\frac{2}{3} + \frac{1}{4} =$	$\frac{2}{3} + \frac{3}{5} =$

Case 2: Denominators With One Common Factors

Factor, Crush, Spotlight

Trap!
If denominators have common factors, direct spotlighting produces overly large equivalent fractions.

Must reduce with a Division Diet!

$$\boxed{\frac{6}{24}}\quad \frac{1}{4} + \frac{1}{6} \quad \boxed{\frac{4}{24}} = \frac{\mathbf{10}}{\mathbf{24}}\begin{matrix}\div 2\\ \div 2\end{matrix} = \frac{5}{12}$$

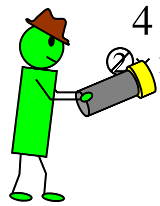
To avoid the Trap, extract prime factors from each

$$\underset{2\times 2}{\frac{1}{4}} + \underset{2\times 3}{\frac{1}{6}}$$

$$\underset{\cancel{2}\times 2}{\frac{1}{4}} + \underset{\cancel{2}\times 3}{\frac{1}{6}}$$

Crush (cross out) common factors.

Spotlight to the right with the uncrushed factor/s.

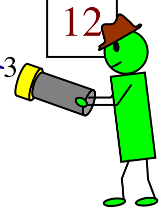


$$\underset{\cancel{2}\times 2}{\frac{1}{4}} + \underset{\cancel{2}\times 3}{\frac{1}{6}} \quad \boxed{\frac{2}{12}}$$

Box the equivalent fraction.

Box the equivalent fraction.

$$\boxed{\frac{3}{12}} \quad \underset{\cancel{2}\times 2}{\frac{1}{4}} + \underset{\cancel{2}\times 3}{\frac{1}{6}} \quad \boxed{\frac{2}{12}}$$



Spotlight to the left with the uncrushed factor/s.

Combine equivalent fractions.

$$\boxed{\frac{3}{12}} \quad \underset{\cancel{2}\times 2}{\frac{1}{4}} + \underset{\cancel{2}\times 3}{\frac{1}{6}} \quad \boxed{\frac{2}{12}} = \frac{5}{12}$$

The result is in lowest terms without having to reduce.

Compact Version

$$\boxed{\frac{3}{12}} \quad \underset{\cancel{2}\times 2}{\frac{1}{4}} + \underset{\cancel{2}\times 3}{\frac{1}{6}} \quad \boxed{\frac{2}{12}} = \frac{5}{12}$$

Imagine powerful spotlights arcing across the night sky!

Spotlighting with a Prime Denominator

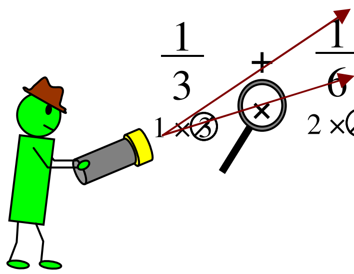
Include a 1 in the factor pair, so you'll have it to spotlight with.

$$\underset{1\times 3}{\frac{1}{3}} + \underset{2\times 3}{\frac{1}{6}}$$

$$\underset{1\times \cancel{3}}{\frac{1}{3}} + \underset{2\times \cancel{3}}{\frac{1}{6}}$$

Crush (cross out) common factors.

Spotlight to the right with the uncrushed factor/s.



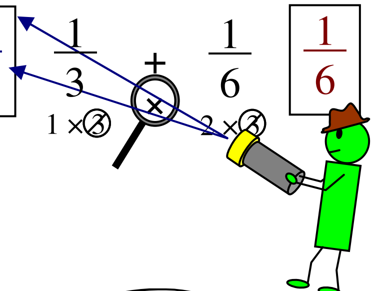
$$\underset{1\times \cancel{3}}{\frac{1}{3}} + \underset{2\times \cancel{3}}{\frac{1}{6}}\quad \boxed{\frac{1}{6}}$$

Box the equivalent fraction.

Observe the fraction is the same.

Box the equivalent fraction.

$$\boxed{\frac{2}{6}}\quad \underset{1\times \cancel{3}}{\frac{1}{3}} + \underset{2\times \cancel{3}}{\frac{1}{6}}\quad \boxed{\frac{1}{6}}$$



Spotlight to the left with the uncrushed factor/s.

Combine equivalent fractions.

$$\boxed{\frac{2}{6}}\quad \underset{1\times 3}{\frac{1}{3}} + \underset{2\times \cancel{3}}{\frac{1}{6}}\quad \boxed{\frac{1}{6}} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

This sum requires a division diet to be in lowest terms.

Your turn: Factor, crush, and spotlight to produce equivalent fractions (no sums this time).

$\boxed{\frac{\ }{\ }}\ \frac{1}{2} + \frac{1}{4}\ \boxed{\frac{\ }{\ }}$	$\boxed{\frac{\ }{\ }}\ \frac{1}{6} + \frac{2}{9}\ \boxed{\frac{\ }{\ }}$
$\boxed{\frac{\ }{\ }}\ \frac{1}{4} + \frac{5}{6}\ \boxed{\frac{\ }{\ }}$	This problem has *multiple* uncrushed factors. $\boxed{\frac{\ }{\ }}\ \frac{1}{18} + \frac{3}{8}\ \boxed{\frac{\ }{\ }}$

Case 3: Denominators With Multiple Common Factors

Factor each denominator into its primes, and crush the common factors.	Spotlight to the left with the remaining factor/s.	Spotlight to the right with the remaining factor/s.
$\frac{1}{8} + \frac{1}{12}$ $2 \times 2 \times 2 \quad 2 \times 2 \times 3$	$\boxed{\frac{3}{24}} \quad \frac{1}{8} + \frac{1}{12}$ $2 \times 2 \times 2 \quad 2 \times 2 \times 3$	$\boxed{\frac{3}{24}} \quad \frac{1}{8} + \frac{1}{12} \quad \boxed{\frac{2}{24}}$ $2 \times 2 \times 2 \quad 2 \times 2 \times 3$

Your turn: Factor, crush, and spotlight to produce equivalent fractions (no sums this time).

Since both 2s will be crushed, include a 1 for spotlighting. $\boxed{\frac{}{}} \quad \frac{1}{4} + \frac{3}{8} \quad \boxed{\frac{}{}}$ $2 \times 2 \times 1 \quad 2 \times 2 \times 2$	$\boxed{\frac{}{}} \quad \frac{3}{8} + \frac{5}{12} \quad \boxed{\frac{}{}}$
$\boxed{\frac{}{}} \quad \frac{1}{12} + \frac{1}{18} \quad \boxed{\frac{}{}}$	You'll have *three* factors to crush in this problem! $\boxed{\frac{}{}} \quad \frac{1}{16} + \frac{1}{24} \quad \boxed{\frac{}{}}$

Your turn: Factor, crush, and spotlight to produce equivalent fractions and sums.
Reduce if needed. The Answer Key will show only the sums in lowest terms.

$\frac{1}{5} + \frac{3}{10} =$	$\frac{1}{6} + \frac{5}{12} =$
$\frac{2}{7} + \frac{3}{14} =$	$\frac{5}{9} + \frac{5}{6} =$

Adding Mixed Numbers

When adding mixed numbers, you have more than one option.

All Improper Option Convert the mixed numbers to improper fractions, then add.	**Separate Sums Option** Add the whole numbers and fractions separately; then combine the sums.
$2\frac{1}{5} + 1\frac{1}{3}$ 10 + 3 + $2\frac{1}{5} + 1\frac{1}{3}$ $\frac{11}{5} + \frac{4}{3}$ $\boxed{\frac{33}{15}}\ \frac{11}{5} + \frac{4}{3}\ \boxed{\frac{20}{15}}$ $\frac{53}{15}$ Convert to mixed if desired. $15 \overline{)53}$ = $3\frac{8}{15}$ 45 8 8	$2\frac{1}{5} + 1\frac{1}{3}$ $(2 + 1)\ \left(\frac{1}{5} + \frac{1}{3}\right)$ $3 \quad \boxed{\frac{3}{15}}\ \frac{1}{5} + \frac{1}{3}\ \boxed{\frac{5}{15}} = \frac{8}{15}$ $3\frac{8}{15}$ Convert to improper if desired. 45 + $3\frac{8}{15} = \frac{53}{15}$

Your turn: On scratch paper, add the mixed numbers using both options.

All Improper Option   Separate Sums Option

$$2\frac{1}{2} + 3\frac{1}{5} = \quad \bigcirc \quad \bigcirc$$

Mixed-to-Improper Half Spotlighting

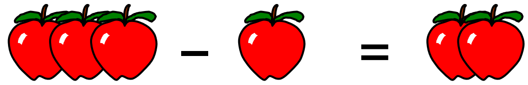
A mixed number is an addition, e.g., 1½ = "1 and ½" = 1 + ½. The mixed-to-improper conversion trick works by half spotlighting (see p.24). Proof: Place the whole number over a 1 and do a full spotlight.

$$1\frac{1}{2} = 1 + \frac{1}{2} = \frac{1}{1} + \frac{1}{2} = \boxed{\frac{2}{2}}\ \frac{1}{1} + \frac{1}{2}\ \boxed{\frac{1}{2}} = \frac{3}{2}$$

Subtracting Fractions

Subtracting Like Fractions

'Like' fractions have identical denominators.

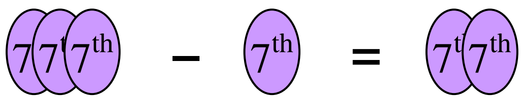


3 apples – 1 apple = 2 apples

It's easy to subtract apples.

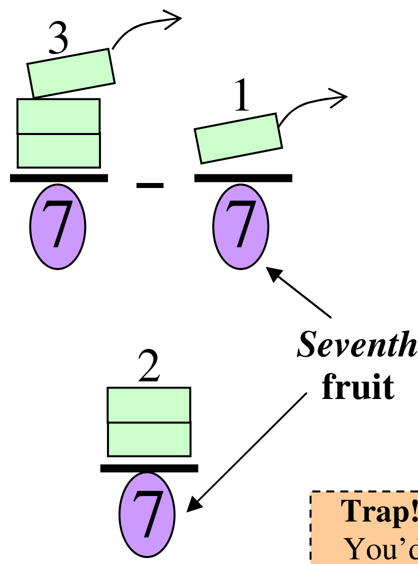
BrainAid
Imagine denominators are fruit. 'Like' fractions have the *same* fruit on the bottom.

If you had a different kind of fruit called "sevenths," they'd be just as easy to subtract.



3 sevenths – 1 seventh = 2 sevenths

To subtract 'like' fractions, steal an equal amount from each numerator and place the difference over one denominator.



Trap! Do *not* subtract denominators!
You'd have zero fruit on the bottom!

$$\frac{3}{7} - \frac{1}{7} \neq \frac{2}{0}$$

Your turn: Subtract the following like-denominator fractions.

$\frac{3}{8} - \frac{2}{8} =$	$\frac{5}{6} - \frac{4}{6} =$	$\frac{6}{5} - \frac{2}{5} =$
$\frac{7}{8} - \frac{4}{8} =$	$\frac{2}{7} - \frac{1}{7} =$	$\frac{7}{9} - \frac{5}{9} =$

Subtracting Unlike Fractions

You can not directly subtract fractions with different denominators. You must use Spotlighting (see p.33) to create equivalent fractions with the Least Common Denominator.

Factor, Crush, Spotlight & Subtract

$$\frac{1}{4} - \frac{1}{6}$$

$$\boxed{\frac{3}{12}} \quad \frac{1}{4} - \frac{1}{6} \quad \boxed{\frac{2}{12}} = \frac{1}{12}$$

$$\cancel{2} \times 2 \qquad \cancel{2} \times 3$$

Your turn: Factor, crush, spotlight, subtract, and reduce as needed.
The Answer Key will show only the final answers in lowest terms.

$\frac{3}{10} - \frac{1}{5} =$	$\frac{3}{4} - \frac{2}{5} =$
$\frac{5}{6} - \frac{3}{8} =$	$\frac{5}{12} - \frac{5}{18} =$
$\frac{1}{6} - \frac{1}{16} =$ 16 has *three* uncrushed factors to spotlight.	$\frac{5}{8} - \frac{5}{14} =$

Subtracting Mixed Numbers

When subtracting mixed numbers, you have more than one option.

All Improper Option Convert the mixed numbers to improper fractions, then subtract.	**Separate Differences Option** Subtract the whole numbers and fractions separately; then combine the differences.
$3\frac{1}{3} - 1\frac{1}{4}$	$3\frac{1}{3} - 1\frac{1}{4}$
9 + 4 + $3\frac{1}{3} - 1\frac{1}{4}$	$\left(3 - 1\right)\left(\frac{1}{3} - \frac{1}{4}\right)$
$\frac{10}{3} - \frac{5}{4}$	2
$\boxed{\frac{40}{12}}\ \frac{10}{3} - \frac{5}{4}\ \boxed{\frac{15}{12}}$	$\boxed{\frac{4}{12}}\ \frac{1}{3} - \frac{1}{4}\ \boxed{\frac{3}{12}} = \frac{1}{12}$
$\frac{25}{12}$	$2\frac{1}{12}$
Convert to mixed if desired.	Convert to improper if desired.
$12 \overline{)\,25}$ quotient $2\frac{1}{12}$ $\underline{24}$ 1 $\underline{1}$	24 + $2\frac{1}{12} = \frac{25}{12}$

Your turn: On scratch paper, subtract the mixed numbers using both options.

	All Improper Option	Separate Differences Option
$4\frac{1}{2} - 2 =$		
$5\frac{2}{3} - 2\frac{1}{4} =$		

Negative Fraction Issue

When subtracting mixed numbers using the Separate Differences Option, if the fraction of the first mixed number is smaller than the fraction of the second mixed number, the result is a negative fraction. The traditional way to avoid having to deal with a negative fraction is the Borrow Option. Alternately, you could use the All Improper Option to avoid a negative situation altogether.

Borrow Option

Borrow a 1 from the first whole number to make its fraction larger than the second fraction.

$$3\frac{1}{3} - 1\frac{1}{2}$$

Split

$$\left(2 + 1 + \frac{1}{3}\right) - 1\frac{1}{2}$$

Convert

$$\left(2 + \frac{3}{3} + \frac{1}{3}\right) - 1\frac{1}{2}$$

Add

$$2 + \frac{4}{3} - 1\frac{1}{2}$$

$$\left(2 - 1\right) \qquad \left(\frac{4}{3} - \frac{1}{2}\right)$$

$$1 \qquad \boxed{\frac{8}{6}} \; \frac{4}{3} - \frac{1}{2} \; \boxed{\frac{3}{6}} = \frac{5}{6}$$

$$1\frac{5}{6}$$

Your turn: On scratch paper, subtract the mixed numbers using the Borrow Option.

$4\frac{1}{5} - 2\frac{1}{4} =$ ◯ $\qquad 6\frac{3}{5} - 3\frac{3}{4} =$ ◯

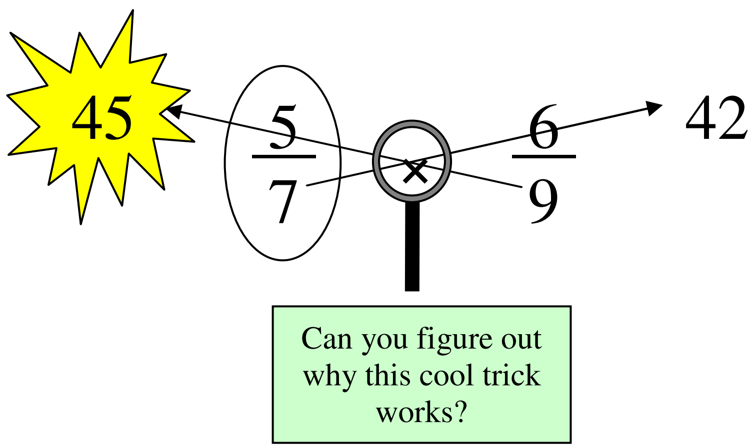
Comparing Fractions: Spotlight Tops

You've been working hard, so it's time for a little fun.

Problem: Which fraction is larger? Are you sure? It's not always easy to tell just by looking.

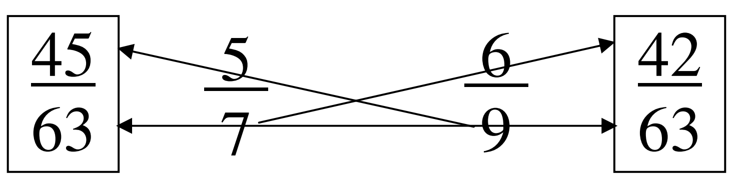
$$\frac{5}{7} \qquad \frac{6}{9}$$

Solution: Multiply the numerator of each fraction by the denominator of the other.
The largest Cross-Product indicates the largest fraction.
In this case, 45 is larger than 42, so 5/7 is larger than 6/9.



Why It Works

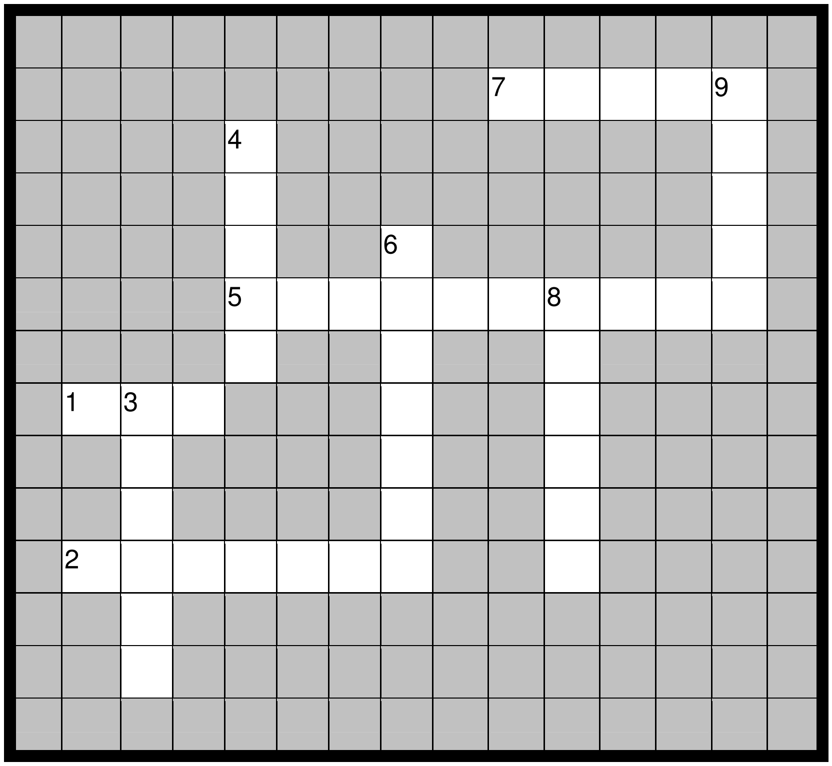
This is Spotlighting without multiplying the denominators, since both equivalent denominators are guaranteed to be the same. Spotlighting the denominators reveals that 45 of 63 parts is greater than 42 of 63.



Your turn: Spotlight the numerators to find the largest fraction. Circle it.

$\frac{6}{7} \quad \frac{5}{6}$	$\frac{4}{9} \quad \frac{5}{11}$	$\frac{7}{6} \quad \frac{9}{8}$

BrainDrain #2



Fill in the Crossword Puzzle

Across

1. The greatest common factor is the _____.
2. Proper x Proper = _____ Product.
5. Reduced fractions are _____ fractions.
7. Equivalent fractions are _____ in value.

Down

3. Added fractions need a _____ denominator.
4. Addition converts _____ to improper.
6. To divide fractions, first invert the _____.
8. Reduce fractions to their _____ terms.
9. The LCD is the _____ common denominator.

True/False

Write T or F in the blanks.

1____ Spotlighting always produces the LCD.

2____ The LCD is the LCM of two denominators.

3____ Factor/crush produces the lowest equivalents.

4____ Compare fraction size by cross multiplying.

5____ You can Cross-Reduce added fractions.

The Tortoise Wins

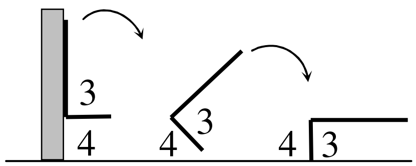
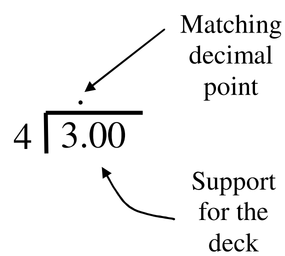
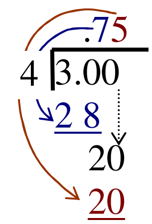
Like the parable of the plodding tortoise winning the race over the swift but overconfident hare, you'll achieve the greatest math accuracy by following a step-by-step procedure rather than making a quick leap to an incorrect answer.

Math Anywhere Anytime!
If you have access to pencil and paper (scraps will do) and a spare moment, make up fraction problems and practice solving them. If you're not sure of your answers, find someone to check them.

Converting Fractions & Decimals

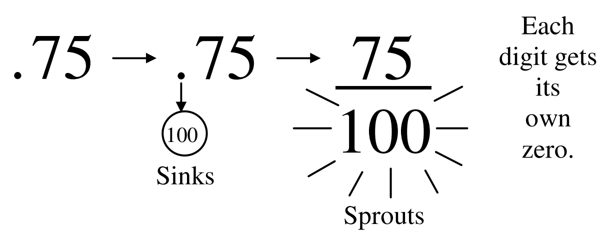
Before you start this section, you may want to review “Decimals = Fractions” on page 10.

Fraction to Decimal: Rack to Deck

Unbolt the fraction rack and rotate it to the floor where it becomes a decimal deck.	Place a decimal point and some zeros under the deck for support. Place a decimal point on top of the deck to match the one below.	Perform Rainbow (aka Long) Division to create the equivalent decimal on top of the deck.
		

Decimal to Fraction: Sink & Sprout

Imagine that decimal points have powers of 10 crammed into them.
They are so dense they sink below ground and expand like a seed until their power of 10 sprouts out.
The power of 10 consists of a 1 followed by as many zeros as there are decimal digits.
Like plants supported by roots, each aboveground digit needs a zero to support it.



Proofs of Equivalence

Spotlight Tops If the Cross-Products of the original and decimal fractions are equal, the fractions are equivalent.	**Reduce** If the decimal fraction reduces to the original fraction, the fractions are equivalent.
$300 \quad \frac{3}{4} \quad \frac{75}{100} \quad 300$	$\frac{75}{100} = \frac{3 \times \cancel{5}^{1} \times \cancel{5}^{1}}{2 \times 2 \times \cancel{5}_{1} \times \cancel{5}_{1}} = \frac{3}{4}$

Your turn: Rack to Deck the following fractions into their decimal equivalents (to two decimal places).

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{2}{3}$	$\frac{3}{5}$	$\frac{4}{7}$

Sink & Sprout & Place Values

Each decimal place adds another zero to the denominator.
BrainAid: The sinking seed contains one zero for each decimal place occupied.

Whole **part**

T h t o • $\frac{1}{t}$ $\frac{1}{h}$ $\frac{1}{T}$ $\frac{1}{tT}$ ten thousandths

ones
tens
hundreds
Thousands
thousandths
hundredths
tenths

Tenths

$.3 \rightarrow \frac{3}{10}$

10
Sinks
Sprouts

Hundredths

$.30 \rightarrow \frac{30}{100}$

100
Sinks
Sprouts

Thousandths

$.300 \rightarrow \frac{300}{1000}$

1000
Sinks
Sprouts

Your turn: Sink & Sprout the following decimals into their equivalent fractions.

$\frac{.3}{}$	.40	.555
.064	.700	.111

Rounding: High Five!

Rounding reduces the number of digits in a number.
Why? To make the number easier to work with.
When? When you don't need to be too precise.

Problem: When you convert some fractions to decimals, the division may continue to more decimal places than you need. But simply dropping the last digit may not produce an accurate enough result.

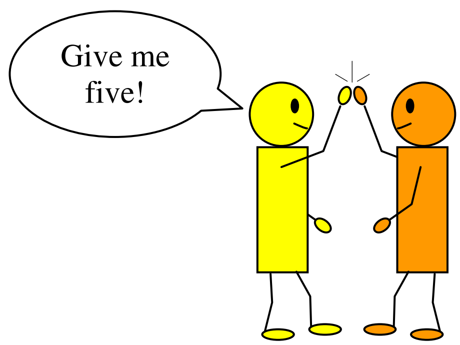
Solution: Round the last (rightmost) digit/s up or down.

Round $^{\text{UP}}$

If the last digit is 5 or higher, delete it and add 1 to the new last digit.
BrainAid: Imagine friends reaching *up* and slapping hands in a "high-five" gesture.

+1

.135 → .135 → .14



Round $_{\text{down}}$

If the last digit is less than 5, delete it.
BrainAid: Imagine numbers less than 5 falling into a hole.

.134 → .134 → .13

Hole

Rounding Multiple Digits

To round to a specific number of digits, draw a box around the unneeded digits. If the boxed number is 50, 500, 5000, etc. or above, round the new last digit up, otherwise just drop the boxed number.

+1

$.21\boxed{500} \rightarrow .22$

Rounding to 2 decimal places.

$.21\boxed{499} \rightarrow .21$

Your turn: Round the following numbers to 2 decimal places.

.748	.352	.4278
.50901	.454999	.5555555

Decimal Operations

Adding & Subtracting Decimals: Align & Sink

To add or subtract decimal numbers, align the decimal points in a column, draw a line beneath, sink a decimal point into the answer area, and add or subtract as usual.

Why it works: Each decimal place has a power-of-10 common denominator.

Align **Add**

$$.3 + .4 \longrightarrow \begin{array}{r} .3 \\ \underline{+\ .4} \end{array} \longrightarrow \begin{array}{r} .3 \\ \underline{+\ .4} \end{array} \longrightarrow \begin{array}{r} .3 \\ \underline{+\ .4} \\ .7 \end{array}$$

Sink

$$\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$

Align **Subtract**

$$3.4 - 1.2 \longrightarrow \begin{array}{r} 3.4 \\ \underline{-\ 1.2} \end{array} \longrightarrow \begin{array}{r} 3.4 \\ \underline{-\ 1.2} \end{array} \longrightarrow \begin{array}{r} 3.4 \\ \underline{-\ 1.2} \\ 2.2 \end{array}$$

Sink

If either number has fewer decimal digits, add zeros as placeholders.

Add zero

$$3.4 + 1.25 \longrightarrow \begin{array}{r} 3.40 \\ \underline{+\ 1.25} \\ 4.65 \end{array} \qquad 3.45 - 1.2 \longrightarrow \begin{array}{r} 3.45 \\ \underline{-\ 1.20} \\ 2.25 \end{array}$$

Add zero

Your turn: Align & Sink, then add or subtract the following decimal numbers.

5.7 + 3.2	5.7 – 3.2
6.2 + 1.9	6.2 – 1.9
12.3+3.25	12.3–3.25

Multiplying Decimals: Match Places

To multiply decimal numbers, multiply the digits as usual,
then match the total decimal places of the multipliers to the product.
Why it works: Each decimal digit creates another zero in the denominator.

Multiply / **Match**

$$\begin{array}{r} .22 \\ \times .3 \\ \hline 66 \end{array} \longrightarrow \begin{array}{r} .22 \\ \times .3 \\ \hline .066 \end{array} \quad \begin{array}{c} \text{2 places} \\ + \\ \text{1 place} \\ = \\ \text{3 places} \end{array}$$

$$\frac{22}{100} \times \frac{3}{10} = \frac{66}{1000}$$

Multiply / **Match**

$$\begin{array}{r} 3.22 \\ \times .42 \\ \hline 644 \\ 1288 \\ \hline 13524 \end{array} \longrightarrow \begin{array}{r} 3.22 \\ \times .42 \\ \hline 644 \\ 1288 \\ \hline 1.3524 \end{array} \quad \begin{array}{c} \text{2 places} \\ + \\ \text{2 places} \\ = \\ \text{4 places} \end{array}$$

Multiplying by Powers of 10: Move Opposite

To multiply a decimal number by a power of 10, make the power of 10 into a 1 by moving its decimal point. Then move the other number's decimal point the same number of places in the *opposite* direction.
Why it works: Moving the decimal points equally in opposite directions creates reciprocals (10 vs. 1/10, 100 vs. 1/100, etc.) that when multiplied yield 1, so no values change, just decimal places.

Move
Left 1 Right 1

$$\begin{array}{ccc} 10 & \times & 4.5 \\ 1 & \times & 45 \\ & 45 & \end{array}$$

$$\frac{1}{10} \times \frac{10}{1} = \frac{1}{1}$$

Move
Left 2 Right 2

$$\begin{array}{ccc} 100 & \times & 4.5 \\ 1 & \times & 450 \\ & 450 & \end{array}$$

Add a zero

Move
Right 2 Left 2

$$\begin{array}{ccc} .01 & \times & 4.5 \\ 1 & \times & .045 \\ & .045 & \end{array}$$

Add a zero

Your turn: Match Places or Move Opposite to multiply the following decimal numbers.

$\begin{array}{r} 3.4 \\ \underline{\times .2} \end{array}$	$\begin{array}{r} 4.43 \\ \underline{\times .2} \end{array}$	$\begin{array}{r} .54 \\ \underline{\times .32} \end{array}$
10×3.2	$.1 \times 3.2$	100×3.2

Dividing Decimals: Drag the Dumbbell

To divide decimal numbers, make the divisor into a whole number by moving its decimal point to the right as far as needed. Move the dividend's decimal point equally to the right. Divide as usual.

Reminder: A whole number like 5 can be written as the mixed decimal 5.0 and vice versa.

Why it works: Moving both decimal points equally is like multiplying the dividend and divisor by the same power of 10. This is equivalent to multiplying by 1, so the value of the division remains the same.

BrainAid: Imagine the two decimal points are the ends of a dumbbell.
Drag the dumbbell to the right until the divisor is an integer, then divide.

Fraction-Division Layout

$$\frac{.42}{.2} = \frac{42}{2} \xrightarrow{\text{Drag right one place}} \frac{42}{2} = \frac{4.2}{2} = 2.1$$

Dividend / Divisor

$$\frac{.42}{.2} \times \overbrace{\frac{10}{10}}^{\text{Equals 1}} = \frac{4.2}{2}$$

Long-Division Layout

$$.2\overline{)\,.42} = 2\overline{)\,42} \xrightarrow{\text{Drag right one place}} 2\overline{)\,42} = 2\overline{)\,4.2}^{\;2.1}$$

Divisor Dividend

If needed, add placeholding zeros to the dividend.

$$.02\overline{)\,.4} = .02\overline{)\,.40} \xrightarrow{\text{Drag right two places}} 2\overline{)\,40} = 2\overline{)\,40}^{\;20}$$

Add a zero

Your turn: Drag the Dumbbell and divide the following decimal numbers.

$\frac{.36}{.3}$	$\frac{.48}{2.4}$ Tip: Use Rack to Deck to divide.
$1.5\overline{)\,3}$	$.03\overline{)\,6}$

Converting Decimals & Percents

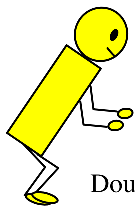
Before you start this section, you may want to review “Percents = Fractions” on page 11.

BrainAid: Let DiP represent Decimal into Percent.

Decimal to Percent: Double DiP Right

Move the decimal point two places *right*
in the D to P direction. Add a % sign.

Decimal ⟶ **Percent**



.05 ⟶ 5 %

Double DiP right. Add a % sign.

Why it works: Any decimal number can be written as a fraction with a 1 in the denominator. Multiplying top and bottom by 100 creates a percent, but this is equivalent to multiplying by 1, so the value of the division remains the same.

$$\frac{.05}{1} \times \overbrace{\frac{100}{100}}^{\text{Equals 1}} = \frac{5}{100} = 5\%$$

Your turn: Double Dip right to convert the following decimals into percents.

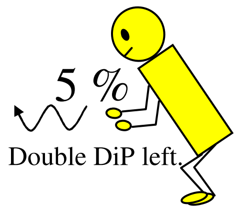
.36 ⟶	3.6 ⟶ Add a placeholding zero.	36 ⟶ Add a decimal point and two zeros.
.0123 ⟶	.123 ⟶	1.23 ⟶

Decimal *from* Percent: Double DiP Left

Move the decimal point two places *left*
in the D direction from P. Delete the % sign.

Decimal ⟵ **Percent**

.05 ⟵ 5 %



Delete the % sign. Double DiP left.

Why it works: Any percent number can be written as a fraction with 100 in the denominator. Dividing top and bottom by 100 creates a decimal, but this is equivalent to dividing by 1, so the value of the division remains the same.

Equals 1

$$5\% = \frac{5}{100} \div \frac{100}{100} = \frac{.05}{1}$$

Your turn: Double Dip left to convert the following percents into decimals.

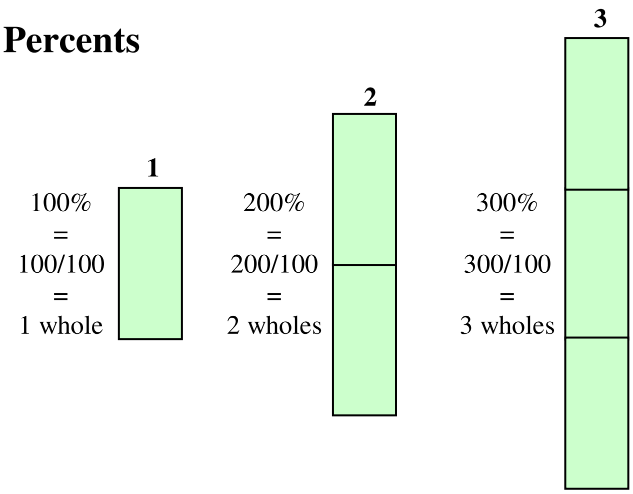
← 3%	← 30%	← 300%
← .25%	← 2.5%	← 25%

The Language of Percents

Sometimes the words and concepts of percents may not match our perceptions.

Whole Number Percents

100% = 1
200% = 2
300% = 3
...
1000% = 10
2000% = 20
3000% = 30

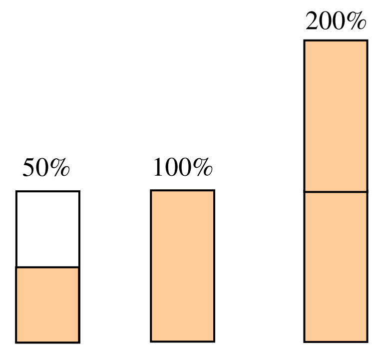


Percent Of

50% *of* an amount means *half* of the original amount.
Example: If someone gave you 50% of $100, you'd have $50.

100% *of* an amount means *one* times the original amount.
Example: If someone gave you 100% of $100, you'd have $100.

200% *of* an amount means *two* times the original amount.
Example: If someone gave you 200% of $100, you'd have $200.



Formula: (Percent of) / 100 = # times the original amount.
Examples: 70% of = 70/100 = 0.7 times the original. 300% of = 300/100 = 3 times the original.

Percent Increase

A 50% *increase* means the original plus half the original, i.e., *1½* times the original amount.
Example: If you increased $100 by 50%, you'd have $100 + $50 = $150.

A 100% *increase* means the original plus one times the original, i.e., *two* times the original amount.
Example: If you increased $100 by 100%, you'd have $100 + $100 = $200.

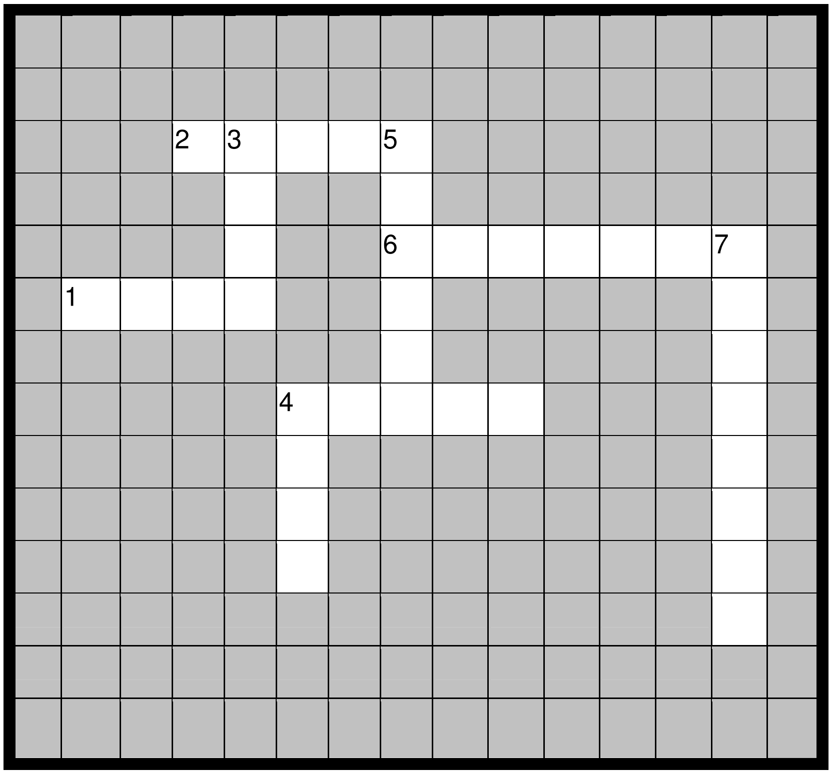
A 200% *increase* means the original plus twice the original, i.e., *three* times the original amount.
Example: If you increased $100 by 200%, you'd have $100 + $200 = $300.

Formula: (Percent increase + 100) / 100 = # times the original.
Examples: 70% increase = (70 + 100)/100 = 170/100 = 1.7 times the original amount.
300% increase = (300 + 100)/100 = 400/100 = 4 times the original amount.

Your turn: Starting with $1000, fill in the blanks with the amount you'd receive.

50% of =	100% of =	200% of =	500% of =
50% increase =	100% increase =	200% increase =	500% increase =

BrainDrain #3



Fill in the Crossword Puzzle

Across	Down
1. Add decimal numbers with Align & _____.	3. Convert fractions to decimals with _____ to Deck.
2. Equivalent fractions have equal _____-products.	4. To multiply by powers of 10, _____ Opposite.
4. To multiply decimal numbers, _____ Places.	5. Convert decimals to fractions with Sink & _____.
6. A rightmost digit of 5 or more is _____ up.	7. To divide decimal numbers, Drag the _____.

True/False

Write T or F in the blanks.

1____ To convert decimal to percent, Double DiP right.

2____ To convert to decimal from percent, Double DiP left.

3____ 200% of an amount is triple the amount.

4____ A 200% increase means triple the original amount.

5____ 300% is equal to 3.

Math Is Easy?

Well, not always. But problems that once seemed impenetrable will fall apart when you “see the light.” Tips, tricks, and analogies can hasten that “breakthrough” moment.

Check of Reasonableness

Once you solve a problem, check to see that the answer you got is reasonable. You can catch many math errors this way. For example, 0.2 x 3 = 6 is not reasonable because 0.2 is less than 1, that is, it’s a part of a whole. Therefore, part (0.2) of 3 has to be less than 3. The correct answer is 0.6.

Answer Key

BrainDrain #1

Page 13

Crossword Puzzle: Across: 1. mixed, 2. denominator, 3. improper, 5. fraction;
Down: 4. percent, 6. numerator, 7. decimal, 8. part, 9. proper
True/False: All are true.

Equivalent Fractions

Page 15: Making Multiples
Top Row: 12, 16, 20; Bottom Row: 15, 20, 25; (20s circled)

Page 16: Making Equivalent Fractions
Top Row: 6/9, 8/12, 10/15; Bottom Row: 9/12, 12/16, 15/20 (12s circled)

Page 19: Finding the GCF
3, 10, No GCF (except for 1)

Page 20: Reduce with GCF
3, 4/5; 4, 4/5; 9, 2/3

Page 21: Reduce with Numerator
Top Row: 1/5, 1/4, 1/7; Bottom Row: 1/6, 1/3, 1/10

Page 22: Reduce with Prime Factors
7/10, 2/3

Page 23: Divide Improper to Get Mixed
Top Row: 1 1/3, 1 2/3, 2 1/3; Bottom Row: 1 1/2, 1 1/5, 3 2/3

Page 24: Add Mixed to Get Improper
Top Row: 11/5, 11/4, 10/3; Bottom Row: 11/3, 13/3, 30/7

Multiplying Fractions

Page 266: Proper Multiplier = Smaller Product
1/12, 4/15, 10/21

Page 26: Melt Before Magnifying
Top Row: 3/14, 1/6, 1/3; Bottom Row: 8/15, 1/6, 1/5

Page 27: Improper Multiplier = Larger Product
Top Row: 15/8, 1 1/2, 1 1/4, 1 7/8 | 20/9, 1 1/3, 1 2/3, 2 2/9 | 32/15, 1 3/5, 1 1/3, 2 2/15
Bottom Row: 5/12, 1 2/3 | 7/10, 1 2/5 | 8/9, 1 1/3

Dividing Fractions

Page 28: Diving Divisor
Top: 8/9; Middle: 5/3; Bottom: ½

Page 29: Down-Under Divisor
1/2, 25/16

Page 30: Fractional Division Issues
3, 3/25

Adding Fractions

Page 31: Adding With Like Denominators
Top Row: 3/4, 5/5=1, 6/7; Bottom Row: 4/8=1/2, 6/6=1, 11/2

Page 33: Spotlighting with no Common Factors
Top Row: 7/6, 7/12; Bottom Row: 11/12, 19/15

Page 35: Spotlighting with One Common Factor
Top Row: 2/4, 1/4 | 3/18, 4/18; Bottom Row: 3/12, 10/12 | 4/72, 27/72

Page 36: Spotlighting with Multiple Common Factors
Top Row: 2/8, 3/8 | 9/24, 10/24; Bottom Row: 3/36, 2/36 | 3/48, 2/48
Top Row: 1/2 | 7/12; Bottom Row: 1/2 | 25/18

Page 37: Adding Mixed Numbers
57/10, 5 7/10

Subtracting Fractions

Page 38: Subtracting With Like Denominators
Top Row: 1/8, 1/6, 4/5; Bottom Row: 3/8, 1/7, 2/9

Page 39: Subtracting With Unlike Denominators
Top Row: 1/10, 7/20; Middle Row: 11/24, 5/36; Bottom Row: 5/48, 15/56

Page 40: Subtracting Mixed Numbers
Top Row: 13/6, 2 1/6; Bottom Row: 41/12, 3 5/12

Page 41: Negative Fraction Issue
1 19/20, 2 17/20

Comparing Fractions

Page 42: Cross Multiply
6/7, 5/11, 7/6

BrainDrain #2

Page 43
Crossword Puzzle: Across: 1. GCF 2. Smaller, 5. equivalent, 7. equal;
Down: 3. common, 4. mixed, 6. divisor, 8. lowest, 9. Least
True/False: 1F, 2T, 3T, 4T, 5F

Converting Fractions & Decimals

Page 45: Fraction to Decimal: Rack to Deck
Top Row: .50, .33, .25; Bottom Row: .66, .60, .57

Page 45: Decimal to Fraction: Sink & Sprout
Top Row: 3/10, 40/100, 555/1000; Bottom Row: 64/1000, 700/1000, 1111/10000

Page 46: Rounding: High Five!
Top Row: .75, .35, .43; Bottom Row: .51, .45, .56

Page 47: Add/Subtract Decimals: Align & Sink
Top Row: 8.9, 2.5; Middle Row: 8.1, 4.3; Bottom Row: 15.55, 9.05

Page 48: Multiply Decimals: Match Places / Move Opposite
Top Row: .68, .886, .1728; Bottom Row: 32, .32, 320

Page 49: Dividing Decimals: Drag the Dumbbell
Top Row: 1.2, .2; Bottom Row: 2, 200

Converting Decimals & Percents

Page 50: Decimal to Percent: Double DiP Right
Top Row: 36%, 360%, 3600%; Bottom Row: 1.23%, 12.3%, 123%

Page 51: Decimal from Percent: Double DiP Left
Top Row: .03, .3, 3; Bottom Row: .0025, .025, .25

Page 52: Percent Of / Percent Increase
Top Row: $500, $1000, $2000, $5000
Bottom Row: $1500, $2000, $3000, $6000

BrainDrain #3

Page 53
Crossword Puzzle: Across: 1. Sink, 2. cross, 4. Match, 6. rounded;
Down: 3. Rack, 4. Move, 5. Sprout, 7. Dumbbell
True/False: 1T, 2T, 3F, 4T, 5T

Additional Max Learning Math Books

Mental Math

Algebra Antics