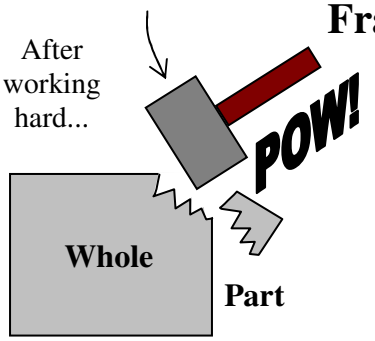
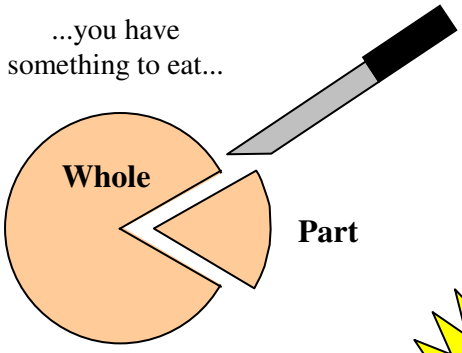
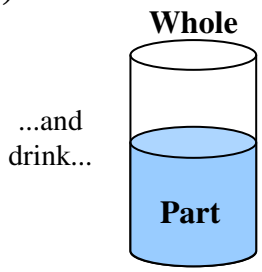
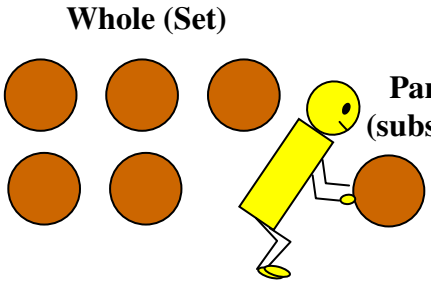
FRACTION FUN!

Fraction = Part of Whole (POW)

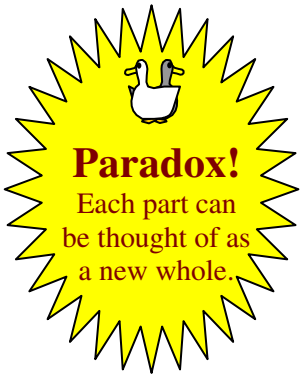




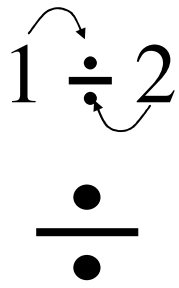




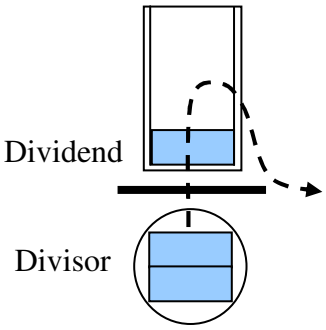
...then play a game!



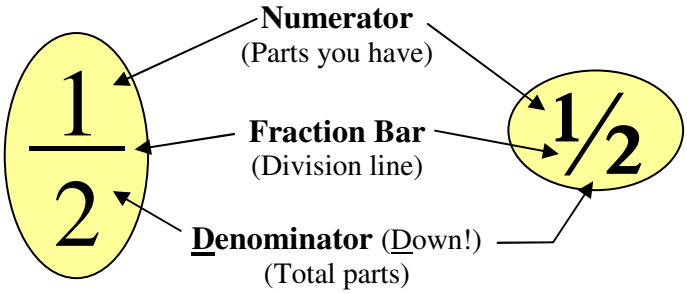
Fraction = Division



The standard division operator looks like a fraction with a number on top and bottom.

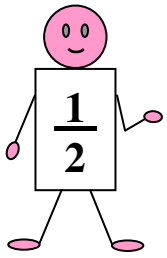


Only ½ of a 2-high tablet (divisor) will dissolve into a 1-high liquid (dividend).

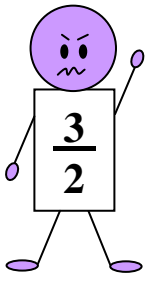


Terms of a Fraction
The numerator and denominator are called the *terms* of the fraction, as in "reduce to lowest *terms*."

Proper Fraction
Behaves as a part.

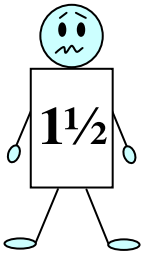


Improper Fraction
Misbehaves: Bigger than a part.



These two can swap personalities and change into each other.

Mixed Number
All mixed up: Whole and part.

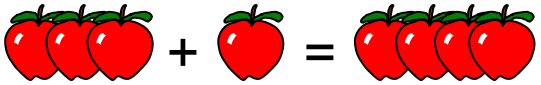


Adding/Subtracting Like Fractions

BrainAid: Imagine denominators are fruit. 'Like' fractions have the *same* fruit on the bottom.

Add

Attach Numerators

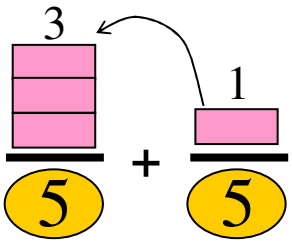


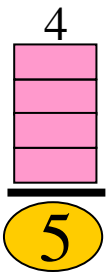
3 apples + 1 apple = 4 apples

Now instead of apples, imagine a tree in the jungle that produces a fruit the natives call a "fifth."

3 fifths + 1 fifth = 4 fifths

To add 'like' fractions, attach the numerators over one denominator.





***Fifth* fruit**

Add
Attach Numerators

$$\frac{1}{3} + \frac{1}{3} = \frac{\quad}{\quad}$$

Draw your choice of shape around each "third" fruit.

TRAP! Do *not* add denominators!
You'd change the type of fruit; e.g.,
3 oranges + 1 orange do *not* make 4 apples!

$$\frac{3}{5} + \frac{1}{5} \neq \frac{4}{10}$$

Subtract

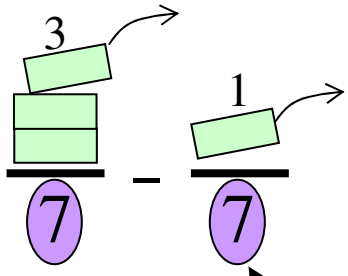
Steal Numerators

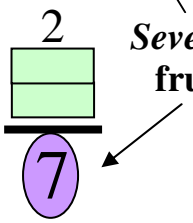
3 apples – 1 apple = 2 apples

Now instead of apples, imagine a lucious purple fruit called a "seventh."

3 sevenths – 1 seventh = 2 sevenths

To subtract 'like' fractions, steal an equal amount from each numerator and place the difference over one denominator.





***Seventh* fruit**

Subtract
Steal Numerators

$$\frac{2}{9} - \frac{1}{9} = \frac{\quad}{\quad}$$

Draw your choice of shape around each "ninth" fruit.

TRAP! Do *not* subtract denominators!
You'd have zero fruit on the bottom; e.g.,
3 oranges – 1 orange do *not* make 0 oranges!

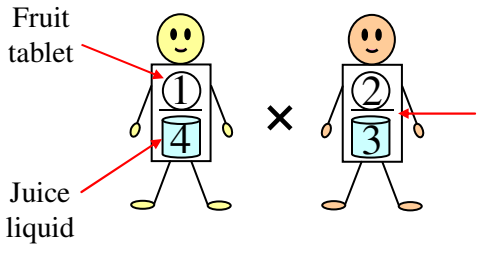
$$\frac{3}{7} - \frac{1}{7} \neq \frac{2}{0}$$

Multiplying Fractions

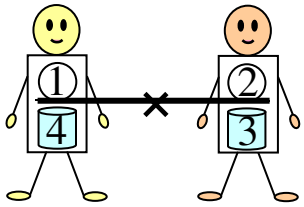
Merge, Melt, & Magnify

BrainAid

Imagine MathBots consume fruit (tablets) and juice (liquid) which can dissolve (melt) through digestive membranes (fraction bars).

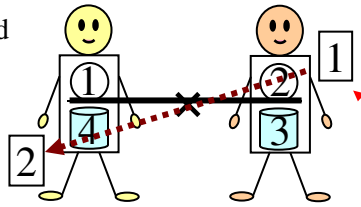


MERGE fraction bars into one long membrane.



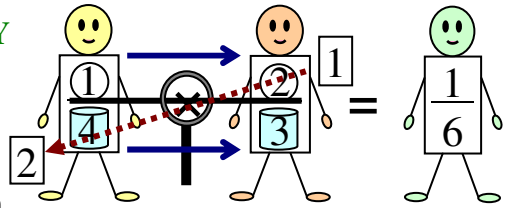
Merged membranes enable *cross*-melting.

MELT using dashed arrows to dissolve tablets into liquids if possible.

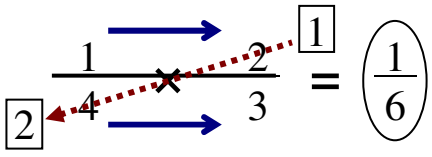


A dissolved tablet always leaves a residue of '1' behind.

MAGNIFY using solid arrows to multiply across top and bottom terms.



Compact Version



Circle the answer.

Tablets and liquids can be on the top or bottom and melt up or down.

Multiply

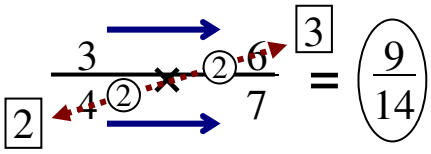
Merge, Melt, & Magnify

$$\frac{3}{4} \times \frac{8}{9} = \text{—}$$

This problem has a double melt!

Antacid Partial Melt

Melting before magnifying avoids the need to reduce later. If a tablet won't completely melt into a liquid, bring in antacid tablets (common factors) to aid digestion and melt both by the same amount.



Multiply

Merge, Melt, & Magnify

$$\frac{5}{9} \times \frac{6}{11} = \text{—}$$

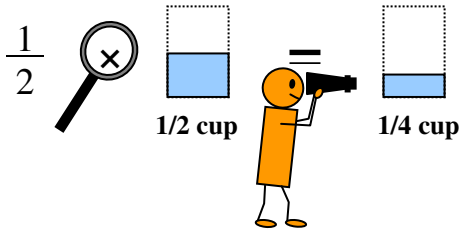
This problem needs antacid tablets!

Paradox!

Magnifying proper fractions creates a *smaller* fraction because you're taking "part of a part."

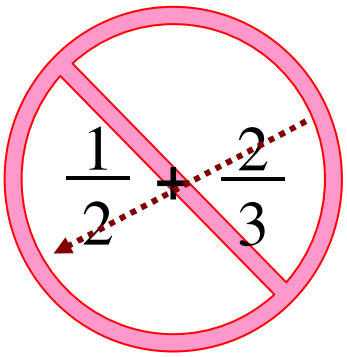
BrainAid

Imagine looking *backwards* through binoculars, magnifying in reverse.



TRAP!

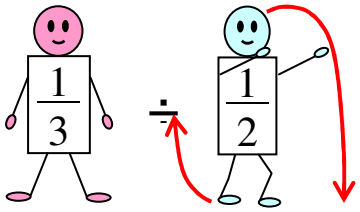
When adding fractions, do *not* merge fraction bar membranes. When adding, cross-melting is *not* allowed!



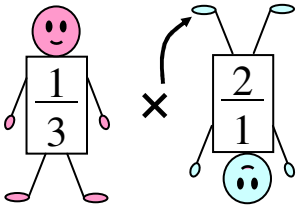
Dividing Fractions

Dive the Divisor

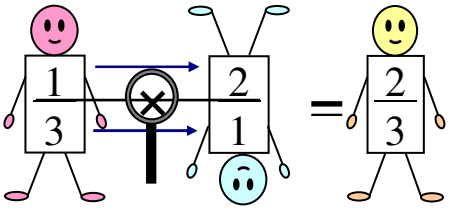
BrainAid: Imagine the divisor diving into a pool.



Diving Divisor



Diver's heel kicks ÷ and flips it to ×.



Merge Melt & Magnify

Divide

Dive the Divisor & Multiply

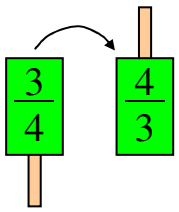
$$\frac{2}{3} \div \frac{5}{7}$$

Inverse = Reciprocal

A flipped fraction is called an inverse or reciprocal [ree-SI-proh-kul].

BrainAid

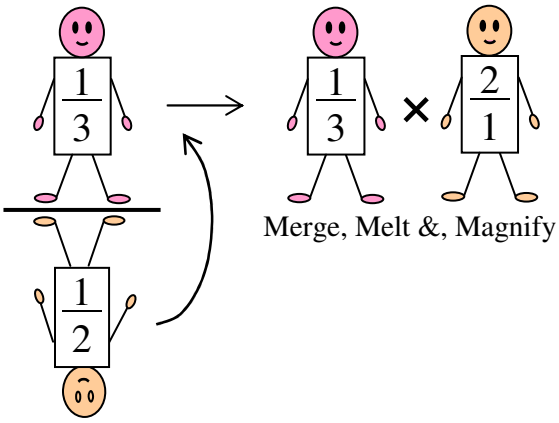
Imagine a reciprocal is a popsicle that flipped over. Think popsicle...re*flip*rocal... reciprocal!



Complex Fraction

Divisor Down Under

It sounds difficult, but a "complex" fraction is just one fraction *vertically* divided by another.



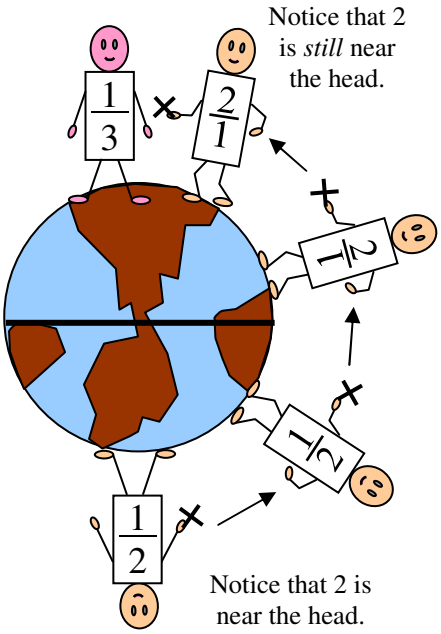
Flip the bottom fraction up to the top.

Merge, Melt &, Magnify

BrainAid

The Divisor Down Under, carrying a replica of the Southern Cross* (multiplication sign) as a gift, walks around the globe, passing the equator (fraction bar) to visit a northern friend.

* The Southern Cross is a star constellation visible from the southern hemisphere.



Notice that 2 is *still* near the head.

Notice that 2 is near the head.

Equivalent Fractions

Equivalent Fractions are equal in value but not in appearance.

Fundamental Property of Fractions
Multiplying or dividing *both* the numerator and denominator by the *same* number creates an equivalent fraction.

– or –

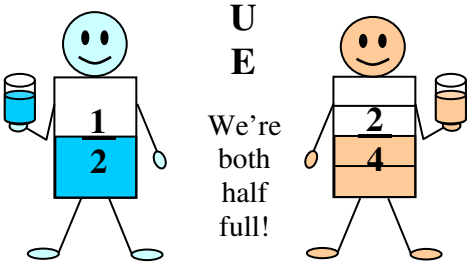
Fun Property of Fractions
Both top and bottom
wanna be hoppin!
Terms just wanna have fun!

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

Equals 1

$$\frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$$

E Q U I V A L E N T
V A L U E

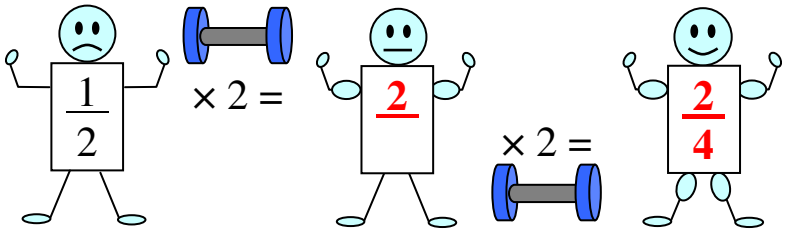


We're both half full!

Why it works.
Multiplying or dividing a number by 1 does not change its value.
½ × 1 = ½
½ ÷ 1 = ½

Multiplying or dividing a fraction by the fractional *equivalent* of 1 changes the fraction's appearance but *not* its value.

Multiply Muscles to Build Fraction



Mr. Thin is unhappy about his puny muscles.

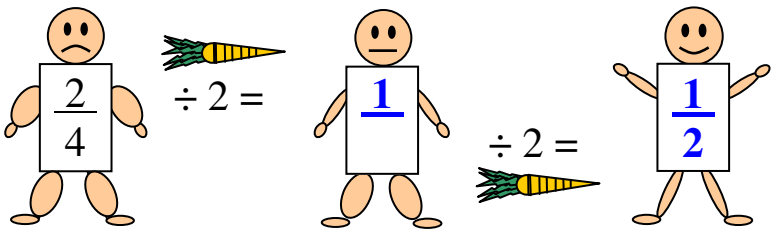
Arm weights *multiply* his upper *muscles,* but now he's top heavy.

Leg weights *multiply* his lower *muscles.* Now he's buff all over and having fun!

Equivalents
Multiply Muscles

$$\frac{1}{2} \times \frac{3}{3} = \frac{\quad}{\quad}$$

Division Diet to Reduce Fraction



Mr. Bulge is unhappy about his weight.

A *division diet* and upper-body aerobics reduce his upper body, but he's still bottom heavy.

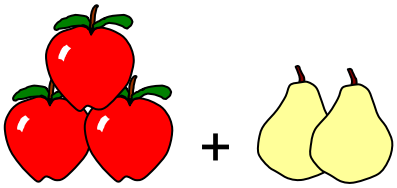
A *division diet* and lower-body aerobics do the trick. Now he's thin and trim all over and having fun!

Equivalents
Division Diet

$$\frac{3}{6} \div \frac{3}{3} = \frac{\quad}{\quad}$$

Unlike Fractions

Fractions with the different fruit on the bottom are called "unlike" fractions.

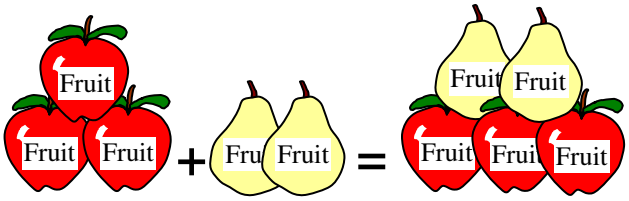


3 apples + 2 pears = ???

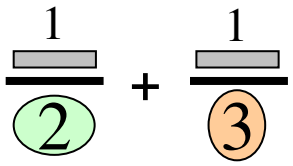
Problem: Can't meaningfully add unlike items like apples and pears.

Question: Do they have anything in common?

Answer: They are all fruits!



3 fruits + 2 fruits = 5 fruits

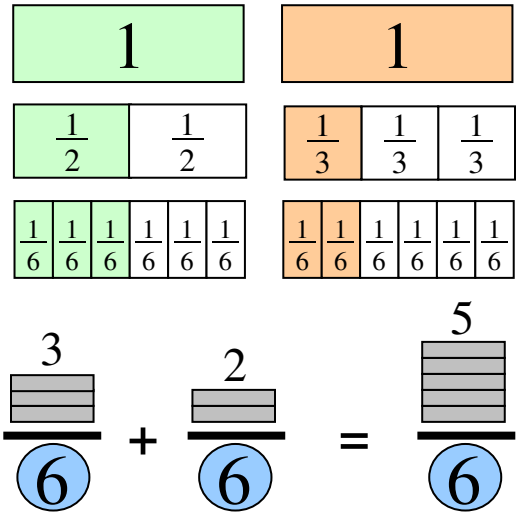


1 half + 1 third = ???

Problem: Can't meaningfully add unlike fractions like halves and thirds.

Question: Do they have anything in common?

Answer: They can both be split into sixths!



3 sixths + 2 sixths = 5 sixths

Question: **How do we make unlike fractions into like fractions?**

Answer: **Create equivalent fractions with common denominators!**

Equivalent Fraction Table

bases	×	2/2	3/3	4/4	5/5	6/6
	1/2	2/4	3/6	4/8	5/10	6/12
	1/3	2/6	3/9	4/12	5/15	6/18

Fraction Series (all equal 1)

Equivalents of 1/2

Equivalents of 1/3

Paradox! The *least* common denominator is *greater* than the original denominators.

Common Denominators: 6, 12

LCD = Least Common Denominator = 6

Using the LCD keeps the equivalent fractions small and easy to work with and the answer at or near to lowest terms.

The LCD is the *smallest* multiple (aka product) that the original denominators will dissolve into.

Spotlighting: Add/Subtract Unlike Fractions

Spotlighting creates equivalent fractions by multiplying the top and bottom of each fraction by the denominator of the other, automatically producing common denominators.

Case 1: Denominators With No Common Factors

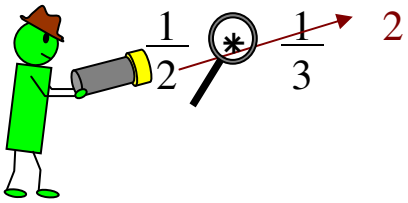
N = Numerator
D = Denominator

$\frac{1}{2} + \frac{1}{3}$

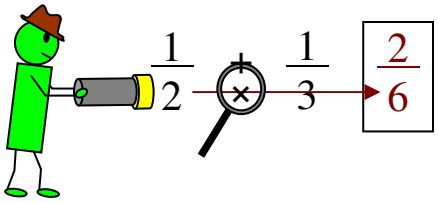
Paradox! To *add* unlike fractions, you must first *multiply*.

Common Factors
Factors are *multipliers* used to make products. Some products have factors in common, e.g., 4 and 6 have a common factor of 2. Other products, e.g., 4 and 9, have no common factors.

Multiply left D times right N.

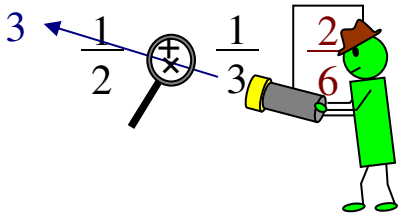


Multiply Ds across bottom.



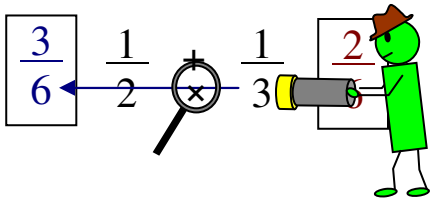
Draw box around right equivalent fraction.

Multiply right D times left N.



Draw box around left equivalent fraction.

Multiply Ds across bottom.



Attach boxed numerators.

Circle the answer.

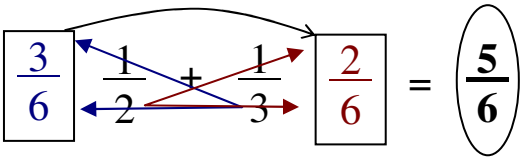
$\frac{3}{6} \frac{1}{2} + \frac{1}{3} \frac{2}{6} = \frac{5}{6}$

SPOTLIGHT SUBTRACT PLUS

BrainAid Spotlight to Subtract or Plus (add) unlike fractions.

Compact Version

Imagine spotlights arcing across the night sky!



Subtract Unlike
Spotlight

$\square\frac{2}{3} - \frac{1}{4}\square = \_\_$

Case 2: Denominators With Common Factors

Factor, Crush, Spotlight

TRAP!
Directly spotlighting denominators that contain common factors produces "overweight" equivalents, e.g., 4 and 6 have a common factor of 2 which doubles both equivalents. The resulting sum then has to be reduced.

Must reduce with a Division Diet!

$\frac{6}{24}$ $\frac{1}{4} + \frac{1}{6}$ $\frac{4}{24} = \frac{\mathbf{10}}{\mathbf{24}} \begin{matrix}\div 2\\ \div 2\end{matrix} = \frac{5}{12}$

$\frac{1}{4} + \frac{1}{6}$

2×2 $\quad 2 \times 3$

To avoid the Trap, first extract prime factors from each denominator.

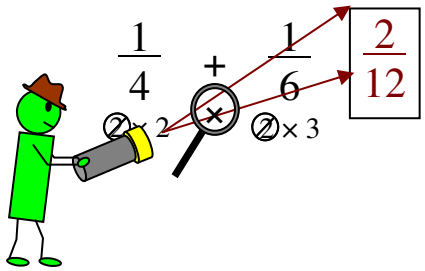
$\frac{1}{4} + \frac{1}{6}$

$\cancel{2} \times 2$ $\quad \cancel{2} \times 3$

Crush (cross out) common factors so no overeating occurs.

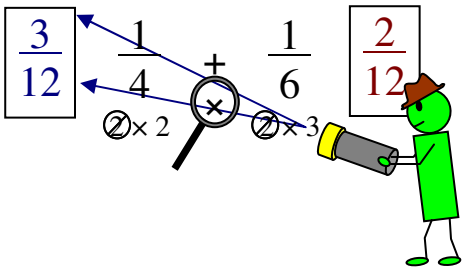
Either method produces the same answer, but (especially) with larger numbers, crushing is less work.

Spotlight to the right with the just uncrushed factor/s.



$\frac{1}{4} + \frac{1}{6} \frac{2}{12}$

$\cancel{2} \times 2$ $\quad \cancel{2} \times 3$



$\frac{3}{12}$ $\frac{1}{4} + \frac{1}{6}$ $\frac{2}{12}$

$\cancel{2} \times 2$ $\quad \cancel{2} \times 3$

Spotlight to the left with just the uncrushed factor/s.

Combine equivalent fractions.

$\frac{3}{12}$ $\frac{1}{4} + \frac{1}{6}$ $\frac{2}{12} = \frac{5}{12}$

$\cancel{2} \times 2$ $\quad \cancel{2} \times 3$

Unless adding numerators creates a sum that has a factor in common with the equivalent denominator, the result will be in lowest terms.

Compact Version

$\frac{3}{12}$ $\frac{1}{4} + \frac{1}{6}$ $\frac{2}{12} = \frac{5}{12}$

$\cancel{2} \times 2$ $\quad \cancel{2} \times 3$

Subtract Unlike

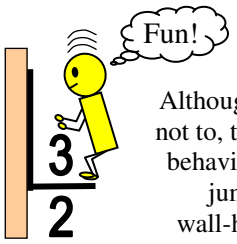
Factor, Crush, Spotlight

$\frac{\square}{\square} \frac{2}{9} - \frac{1}{6} \frac{\square}{\square} = \frac{\square}{\square}$

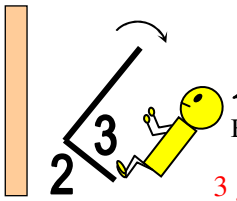
Improper = Mixed

Improper Fractions and Mixed Numbers are alternate forms of the same number.

BrainAid: Imagine an improper fraction on a wall-hinged bed.

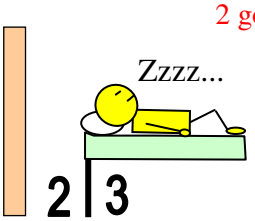


Although being told not to, the Improper-behaving MathBot jumps on a wall-hinged bed.



Bed falls to floor.

3 goes under the bed;
2 goes in front.



Mixed-up MathBot makes bed, goes to sleep, and dreams of being Whole + part.

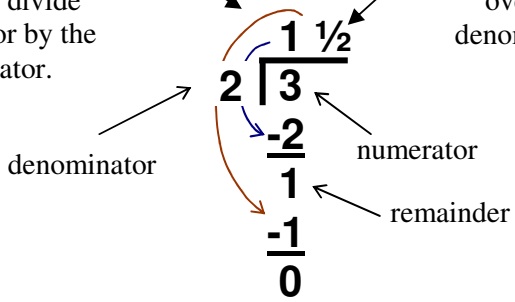
Improper to Mixed

Bed to Floor * Rainbow Division

Mixed denominator is the same as the Improper denominator.

To get the whole number, use Rainbow (aka long) Division to divide the numerator by the denominator.

To get the fraction, place the remainder over the denominator.



Since ½ of 2 is 1, the last (optional) step proves that the fractional part is correct, leaving zero remainder.

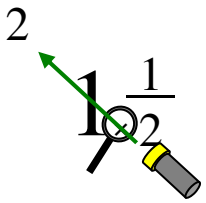
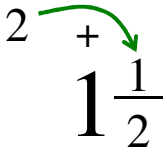
Improper to Mixed

Bed to Floor * Rainbow Division

$\frac{5}{2}$ $\frac{15}{4}$ $\frac{26}{3}$

Mixed to Improper

Half Spotlight & Add

Mixed Number	Half Spotlight (multiply) denominator times whole number.	Add product to numerator.	Place sum over denominator to make an Improper fraction.
$1\frac{1}{2}$			$\frac{3}{2}$

Why It Works

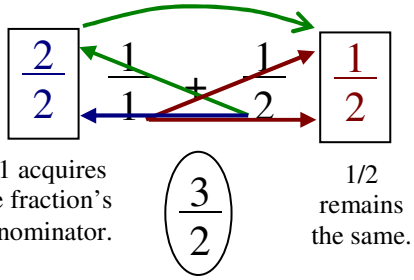
A mixed number is actually an addition, e.g., 1½ is pronounced "1 *and* ½" which means 1 + ½.

$$1\frac{1}{2} = 1 + \frac{1}{2}$$

Change the whole number into a fraction by placing it over a 1.

$$\frac{1}{1} + \frac{1}{2}$$

Spotlight and Add.



1/1 acquires the fraction's denominator.

$\frac{3}{2}$

1/2 remains the same.

Since the denominator remains the same, you only have to 'half' spotlight and add, i.e., follow the **green** arrows only.

Mixed to Improper
Half Spotlight & Add

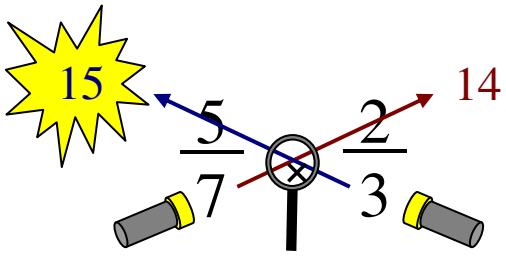
$2\frac{3}{4}$

Comparing Fractions

Top Spotlight

This technique makes it easy to find the larger of two fractions.

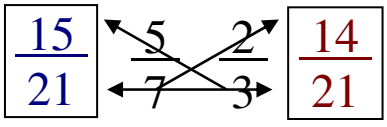
The largest cross-product indicates the largest fraction.



Find the Larger Fraction
Top Spotlight

$\frac{11}{12}$ $\frac{12}{13}$

Question: Why is it *unnecessary* to spotlight across denominators?
Answer: Since spotlighting produces equal denominators, the cross-products alone tell which fraction is larger.



Decimals = Fractions

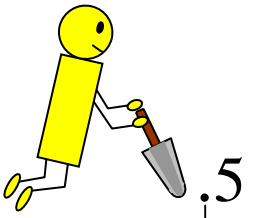
Decimals are fractions whose denominators are powers of 10 (i.e., 10, 100, 1000...).

.5	=	5/10	=	5 ten*ths*
.50	=	50/100	=	50 hundred*ths*
.500	=	500/1000	=	500 thousand*ths*

Decimal to Fraction

Sink & Sprout * Division Diet

Imagine the decimal point is a seed that *sinks* below the ground (fraction bar) and *sprouts* roots.



.5 → 5 / 10 ÷ 5 = 1 / 2 ÷ 5 =

Daisy digit

Put overweight fractions on a Division Diet to reduce them to lowest terms.

Sinks

A straight root sprouts to absorb water.

A circular root sprouts to support *each* daisy digit.

Decimal to Fraction

Sink & Sprout * Division Diet

.25 → ___ ÷ ___ = ___

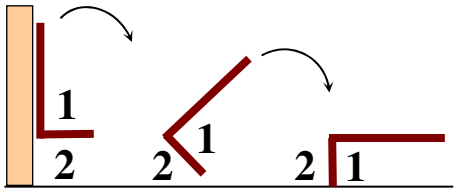
÷ ___ = ___

Fraction to Decimal

Rack to Deck * Rainbow Division

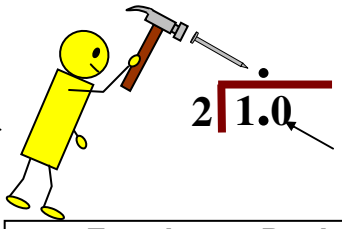
Use Rainbow Division to create the equivalent decimal above the deck.

Imagine that a fraction rack is bolted to a wall.

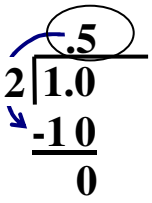


Unbolt and rotate the rack until it becomes a decimal deck.

Hammer deciNAILS (decimal points) above and below to anchor the deck.



Add zero/s to support the deck.



Fraction to Decimal

Rack to Deck * Rainbow Division

1/4

Percents = Fractions

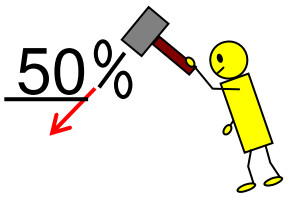
Percents are fractions whose denominators are 100. Per *cent* = Per *100*.

Percent to Fraction:

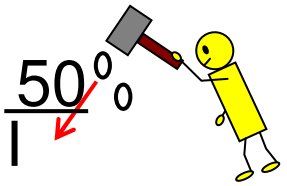
Pound the Percent Sign * Division Diet

50%

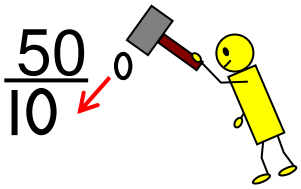
Imagine the % sign is composed of the number 1 sandwiched between a 0 and a 0.



Pound the **/** below the fraction bar where it becomes 1.



Pound each 0 down below the fraction bar.



$\frac{50}{100}$

The % transforms into 100.

Put overweight fractions on a Division Diet.

$\frac{50}{100} \div 50 = \frac{1}{2}$

$\div 50 =$

Percent to Fraction

Pound the Percent Sign * Division Diet

$25\% \longrightarrow \frac{\quad}{\quad} \div \quad = \frac{\quad}{\quad}$

$\div \quad =$

BrainAid

Pound down on a Division *Diet*!

Fraction to Percent:

Multiply Muscles * Heave the Hundredth

$\frac{1}{2}$

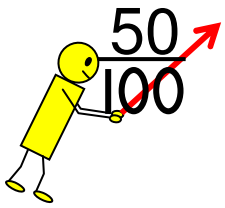
Multiply muscles to make the denominator 100 and the numerator equivalent.

$\frac{1}{2} \times 50 = \frac{50}{100}$

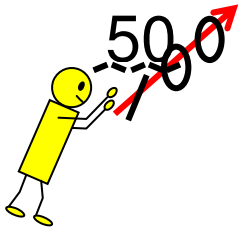
$\times 50 =$

- The original denominator must be a factor of 100 (i.e., 2, 4, 5, 10, 20, 25, or 50). If not, use Rack-to-Deck then Double DiP.
- If the denominator is greater than 100, put it on a Division Diet to reduce it to 100.

Heave the hundredth denominator up through the fraction bar.



The fraction bar shatters.



The 100 transforms into %.

50%

Fraction to Percent

Multiply Muscles * Heave the Hundredth

$\frac{1}{4} \times \quad = \frac{\quad}{\quad} =$

$\times \quad =$

BrainAid

Multiply muscles to *heave* the hundredth.

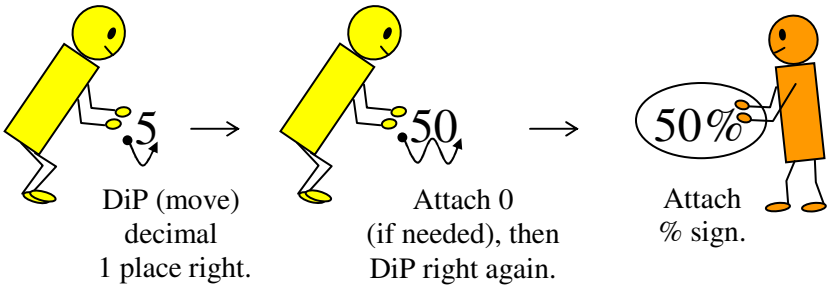
Decimals = Percents

DiP

Decimal into Percent

Decimal *to* Percent

Double DiP Right



DiP (move) decimal 1 place right.

Attach 0 (if needed), then DiP right again.

Attach % sign.

Decimal to Percent
Double DiP Right

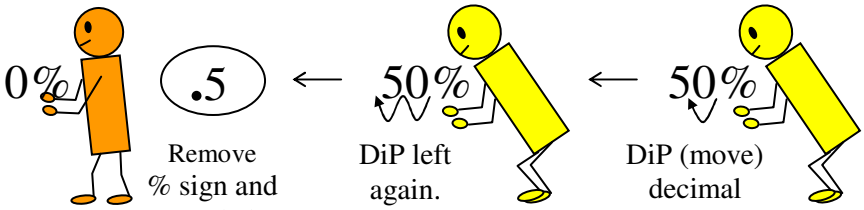
.25 →

Decimal *from* Percent

Double DiP Left

Decimal from Percent
Double DiP Left

← 25%

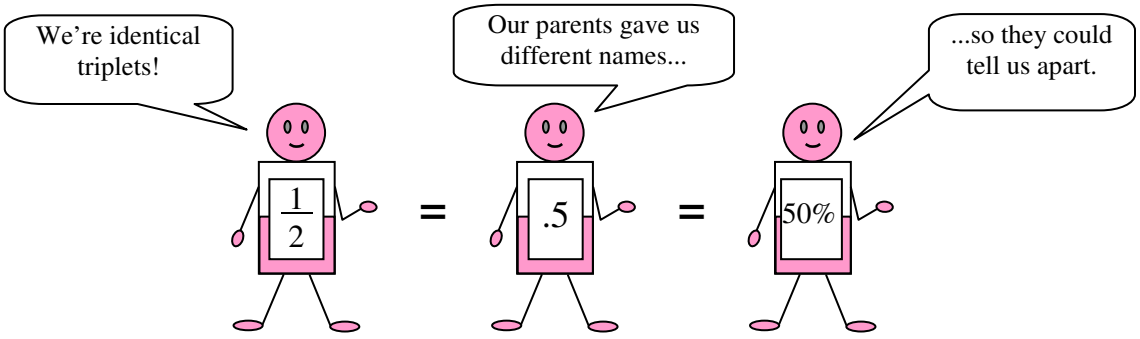


Remove % sign and unneeded 0.

DiP left again.

DiP (move) decimal 1 place left.

Fraction = Decimal = Percent



Why Decimals? They can be easier to calculate than fractions!

$\frac{52}{130} + \frac{76}{152}$

.4 + .5

Why Percents? They can be easier to compare than decimals!

.25 → 25%

.3 → 30%

Fraction Wheel

Fraction = Decimal = Percent

3 numbers × 2 directions = 6 algorithms